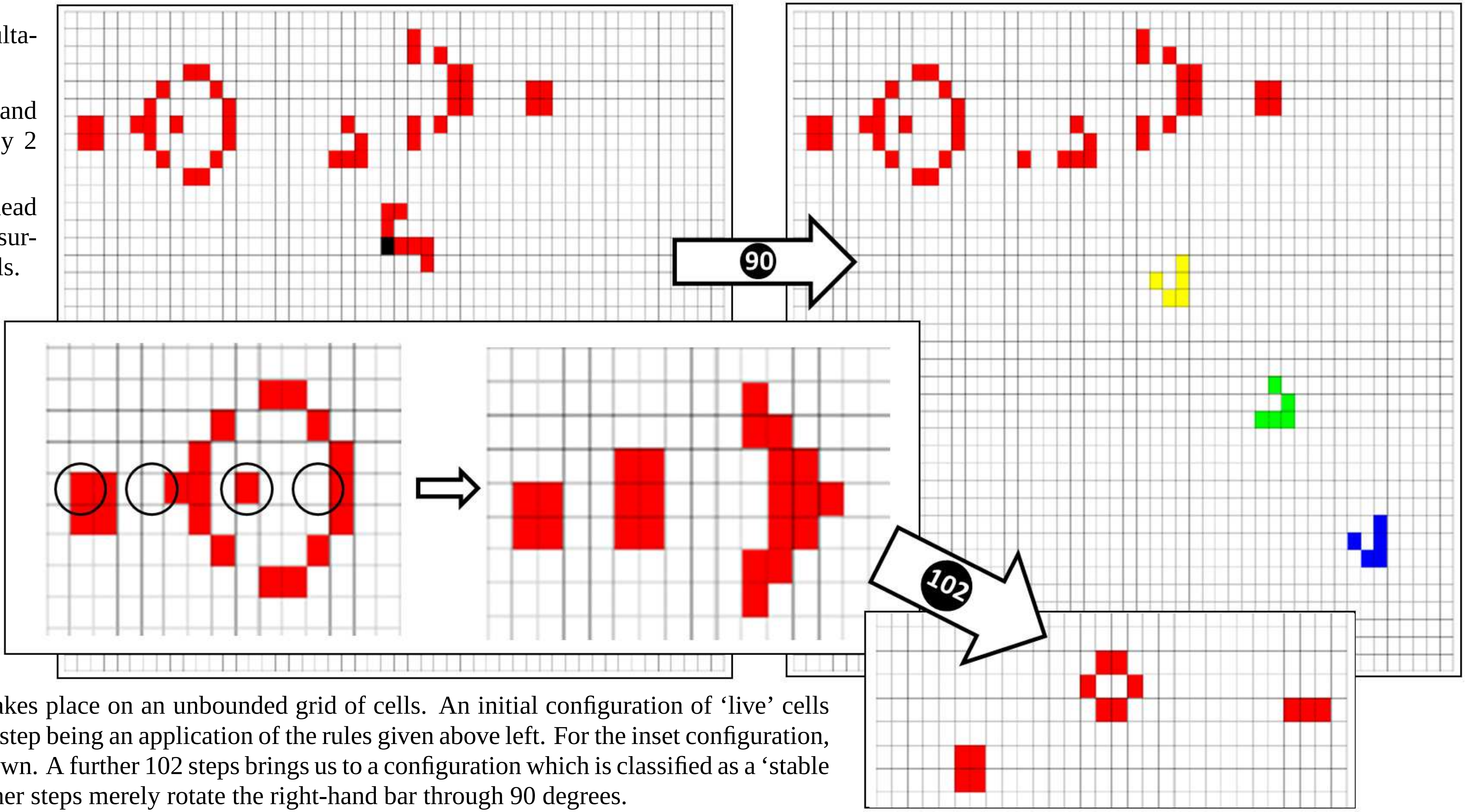
# THEOREM OF THE DAY

**Turing-completeness of Conway's Game of Life** *Conway's 'Game of Life' cellular automaton can be used to simulate any Turing machine.*

**Rules of Life** (Apply simultaneously to all grid cells):

1. A live cell survives if and only if surrounded by 2 or 3 live cells;
2. A dead cell stays dead if and only it is not surrounded by 3 live cells.

The four circled cells on the right illustrate, from left to right, the four possibilities:
Live (red), 3 neighbours: stays live
Dead (blank), 1 neighbour: stays dead
Live, 0 neighbours: dies
Dead, 3 neighbours: becomes live




Conway's Game of Life takes place on an unbounded grid of cells. An initial configuration of 'live' cells evolves step-by-step, each step being an application of the rules given above left. For the inset configuration, one step of evolution is shown. A further 102 steps brings us to a configuration which is classified as a 'stable stationary oscillator': further steps merely rotate the right-hand bar through 90 degrees.

Constructing a working computer (equivalent to a universal Turing machine) in the Game of Life starts with the identification of configurations such as our main image, showing 90 steps of evolution of a 'glider gun'. The colour-coding tries to capture this evolution: the 'gliders' (yellow-green-blue) emerge from the configuration and move diagonally downwards, blue having emerged first. The gun (red cells) oscillates (reloads) with period 30. But, crucially, the black cell in the initial configuration is a 'switch': if it is absent no gliders emerge: the gun is 'off'. Combinations of such guns combine to construct the AND, OR and NOT gates of classical computing.

John Horton Conway introduced the Game of Life in 1970 and published a proof of Turing-completeness in 1982. The glider gun (due to Bill Gosper, 1970) was the first example to be discovered of an indefinitely increasing configuration.

**Web link:** www.alanzucconi.com/2020/10/13/conways-game-of-life/

**Further reading:** *Turing Machine Universality of the Game of Life* by Paul Rendell, Springer, 2016.

Created by Robin Whitty for www.theoremoftheday.org