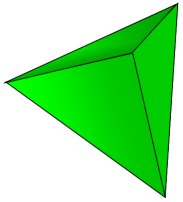
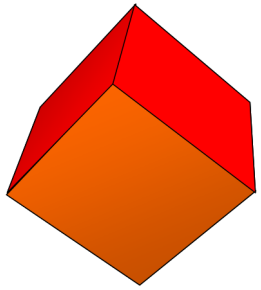
THEOREM OF THE DAY

Theaetetus' Theorem on the Platonic Solids. *There are precisely five regular convex polyhedra, namely the Platonic solids: the tetrahedron, cube, octahedron, icosahedron and dodecahedron.*

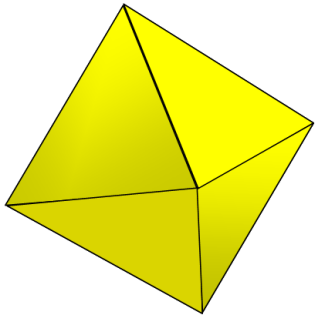


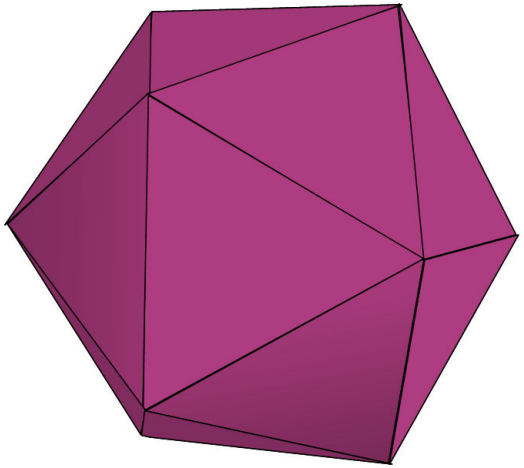


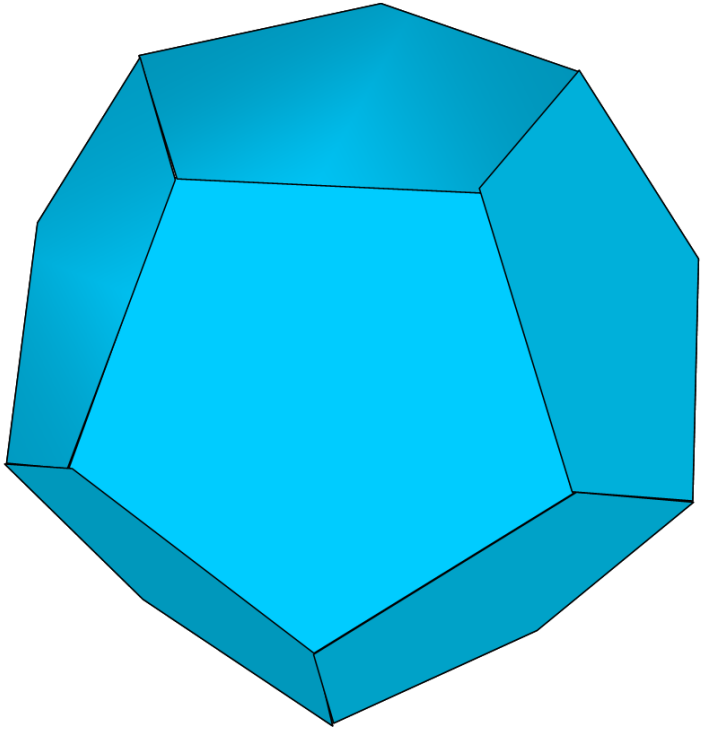
A solid body is *convex* if any straight line joining two of its points is entirely contained within it. It is a *regular polyhedron* if (roughly speaking) it is a volume enclosed by flat faces so that

1. for every vertex, the number and nature of the faces meeting at that vertex, and the angles between the edges incident with the vertex, are all equal, and
2. all faces are congruent regular polygons (all equilateral triangles, all squares, all regular pentagons, etc).

If the second condition is relaxed the body is *semi-regular* and becomes an *Archimedean solid*. It is a remarkable fact that, in three dimensions, the construction of a connected polyhedron, under the conditions of convexity and regularity, excludes all polygons but triangles, squares and pentagons, and that moreover there are just five different constructions possible.







The Platonic solids may have been identified by Pythagoras. Theaetetus of Athens (c. 417-c. 369 BC) gave what is probably the first proof that they are precisely the possible regular convex polyhedra. They are in a sense the climax to Euclid's *Elements* since their theory occupies the whole of the final, thirteenth book.

Web link: www.uwgb.edu/DutchS/SYMMETRY/platonic.htm

Further reading: *Regular Polytopes* by H.S.M. Coxeter, Dover Publications, 1974.