

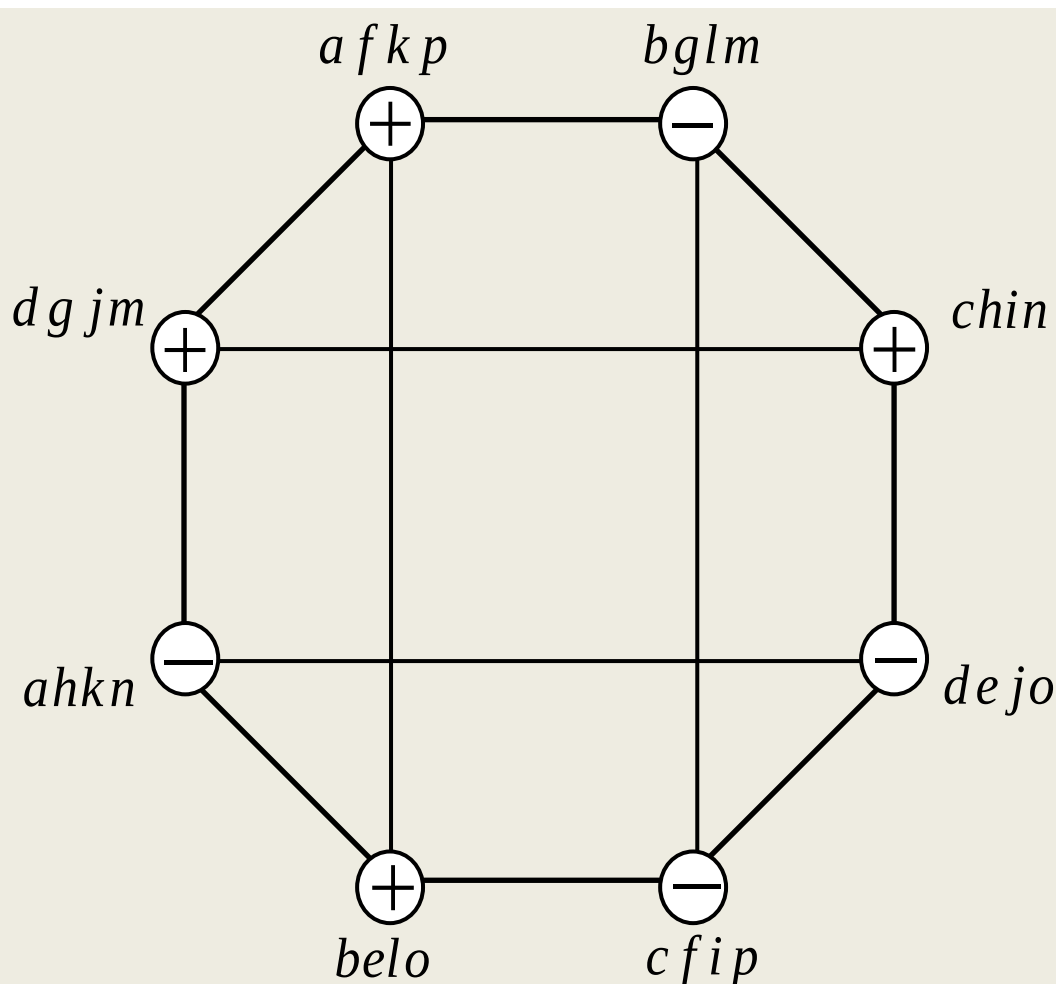
A 4×4 version of the Rule of Sarrus for evaluating determinants

Start by extending a 4×4 matrix as in the Rule of Sarrus. Write the main diagonals; then, in reverse order, the opposite diagonals, with signs as shown. Arrange on the vertices of an octagon. Sum the signed terms. Then add the sums from Rules 1 and 2 as specified.

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}$$



$$\begin{array}{cccc|cccc} a & b & c & d & a & b & c & \\ e & f & g & h & e & f & g & \\ i & j & k & l & i & j & k & \\ m & n & o & p & m & n & o & \\ + & - & + & - & + & - & + & - \end{array}$$



Rule 1

$$\begin{aligned} &+ agln \\ &- bhio \\ &+ cejp \\ &+ dfio \\ &- celn \\ &+ bhkm \\ &- agjp \\ &- dfkm \end{aligned}$$

Rule 2

$$\begin{aligned} &- aflo \\ &- bekp \\ &+ bgip \\ &+ cflm \\ &- dgin \\ &- chjm \\ &+ ahjo \\ &+ dekn \end{aligned}$$

Rule 1: round the octagon take letter 1 of 1st word, then letters 2 and 3 of 2nd word, then letter 4 of 3rd word.

E.g. $+ afkp - bglm + chin = + agln$.

Sign is **opposite** that of middle word.

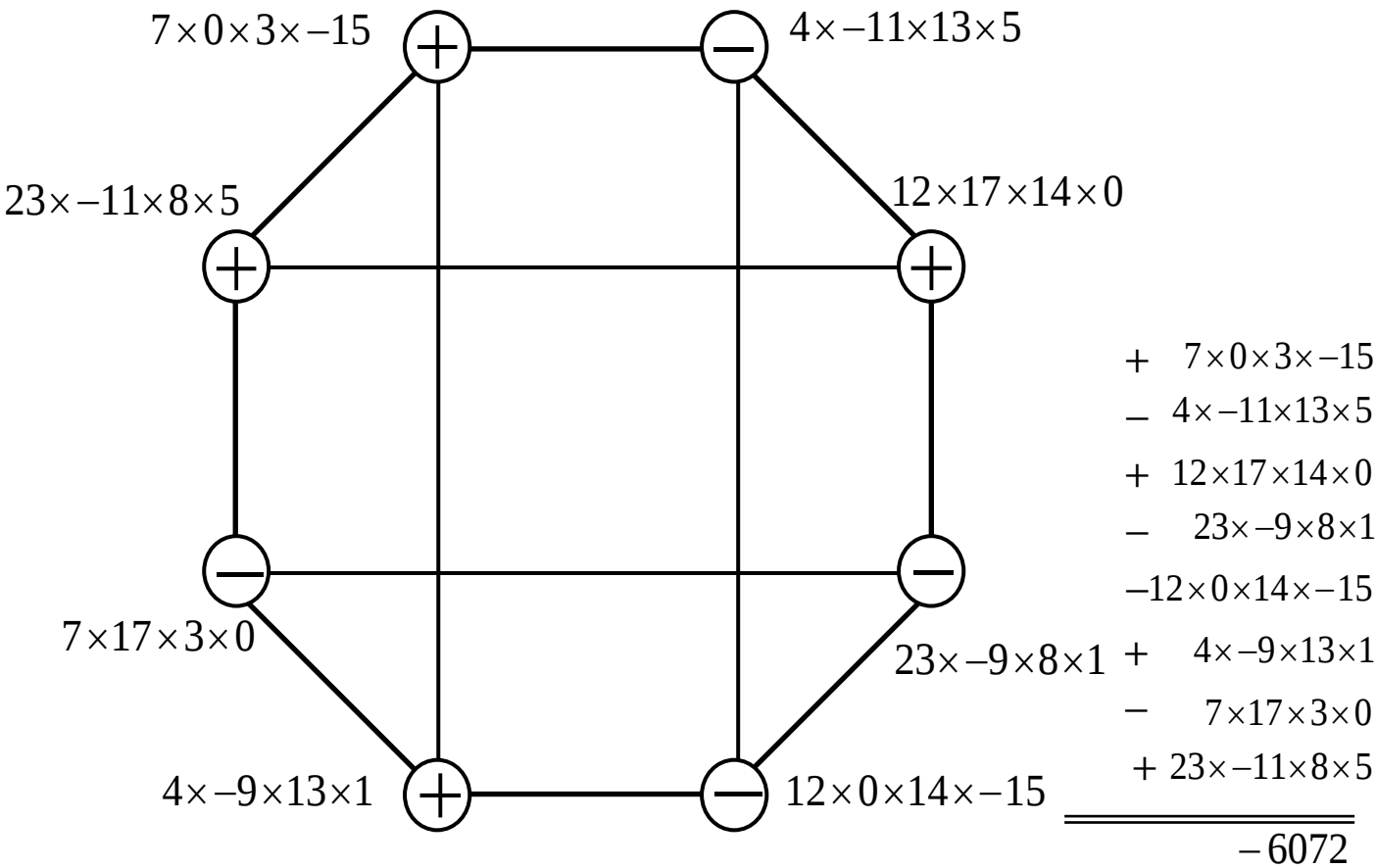
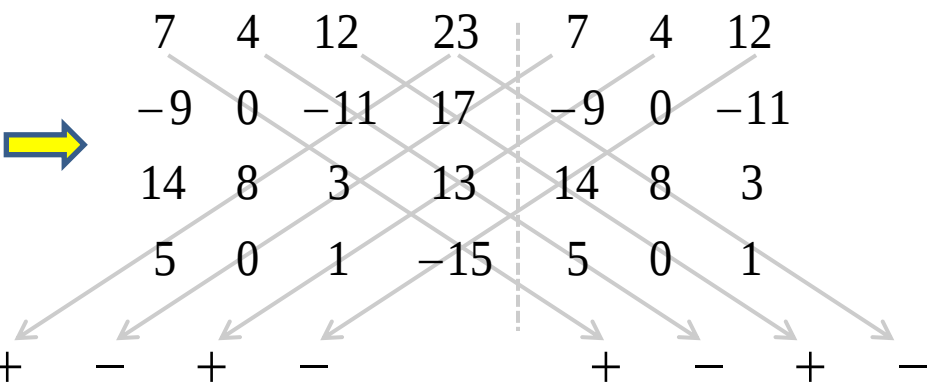
Rule 2: on lines across the octagon take letters 1 and 2 of 1st word then letters 3 and 4 of 2nd word, and vice versa.

E.g. $+ afkp + belo = - aflo$; $+ afkp + belo = - bekp$.

Signs are **opposite** that of joined signs.

An example

$$\begin{vmatrix} 7 & 4 & 12 & 23 \\ -9 & 0 & -11 & 17 \\ 14 & 8 & 3 & 13 \\ 5 & 0 & 1 & -15 \end{vmatrix}$$



Rule 1

$$\begin{aligned} &+ 7 \times -11 \times 13 \times 0 \\ &- 4 \times 17 \times 14 \times 1 \\ &+ 12 \times -9 \times 8 \times -15 \\ &+ 23 \times 0 \times 14 \times 1 \\ &- 12 \times -9 \times 13 \times 0 \\ &+ 4 \times 17 \times 3 \times 5 \\ &- 7 \times -11 \times 8 \times -15 \\ &- 23 \times 0 \times 3 \times 5 \\ \hline &+ 3788 \end{aligned}$$

Rule 2

$$\begin{aligned} &- 7 \times 0 \times 13 \times 1 \\ &- 4 \times -9 \times 3 \times -15 \\ &+ 4 \times -11 \times 14 \times -15 \\ &+ 12 \times 0 \times 13 \times 5 \\ &- 23 \times -11 \times 14 \times 0 \\ &- 12 \times 17 \times 8 \times 5 \\ &+ 7 \times 17 \times 8 \times 1 \\ &+ 23 \times -9 \times 3 \times 0 \\ \hline &+ 412 \end{aligned}$$

Grand total:

$$\begin{aligned} &- 6072 \\ &3788 \\ &412 \\ \hline &- 1872 \end{aligned}$$