**THEOREM OF THE DAY**

Euler’s Continued Fraction Correspondence Let \((a_i)_{i \geq 0}\) be an infinite sequence of nonzero real or complex numbers. Let \(f_n\) denote the \(n\)-th partial sum of the sequence: \(f_n = \sum_{i=0}^{n} a_i\). Then \(f_n\) is also the \(n\)-th convergent of the continued fraction described below, compared for \(n = 1, \ldots, 7\) by plotting the values as shares of a pie.

**Relative error** | \(|(f_n - \tau) / \tau|, (\tau = 2\pi)\) in the \(n\)-th convergent \(f_n\) of the continued fraction described below, compared for \(n = 1, \ldots, 7\) by plotting the values as shares of a pie.

If we apply Euler’s correspondence to Nilakantha’s series with \(a_i = (-1)^{k-1}/k(2k + 1)(2k + 2)\) then we get \(\tau = 6 + \frac{2^2}{12 + \frac{12^2}{6^2 + \frac{12^2}{12 + \frac{6^2}{12^2 + \frac{12^2}{12 + \frac{6^2}{\ldots}}}}}}\), whose convergents are explored in the above pie chart.

Leonhard Euler discovered this correspondence in 1748. The above application to \(\tau\) (in a \(\pi\) version) was given by Douglas Bowman as an alternative derivation of a continued fraction published by Jerome Lange in 1999.


**Further reading:** *Handbook of Continued Fractions for Special Functions* by Annie Cuyt et al, Springer, 2008.