## THEOREM OF THE DAY

Sylvester's Law of Inertia If $Q\left(x_{1}, \ldots x_{n}\right)=a_{11} x_{1}^{2}+\ldots+a_{n n} x_{n}^{2}+2 \sum_{i<j} a_{i j} x_{i} x_{j}$ is any quadratic form with real coefficients then there exists a linear transformation

$$
\begin{aligned}
y_{1} & =\gamma_{11} x_{1}+\ldots+\gamma_{1 n} x_{n}, \\
y_{2} & =\gamma_{21} x_{1}+\ldots+\gamma_{2 n} x_{n}, \\
& \ldots \\
y_{n} & =\gamma_{n 1} x_{1}+\ldots+\gamma_{n n} x_{n},
\end{aligned}
$$

and coefficients $b_{1}, b_{2}, \ldots, b_{n} \in\{0,1,-1\}$, such that $Q\left(x_{1}, \ldots x_{n}\right)=b_{1} y_{1}^{2}+\ldots b_{n} y_{n}^{2}$. Moreover the matrix $\left(\gamma_{i j}\right)$ of coefficients can always be chosen to be nonsingular and then the values $\operatorname{rank}(Q)=\sum\left|b_{i}\right|$ and signature $(Q)=\sum b_{i}$ are uniquely determined, ie. are invariants of $Q$.

$$
\mathrm{Q}\left(x_{1}, x_{2}, x_{3}\right)=3 x_{1}^{2}+10 x_{2}^{2}-5 x_{3}^{2}+12 x_{1} x_{2}-6 x_{1} x_{3}-4 x_{2} x_{3}
$$

$A:\left(\begin{array}{ccc}3 & 6 & -3 \\ 6 & 10 & -2 \\ -3 & -2 & -5\end{array}\right)$

