THEOREM OF THE DAY

Lin McMullin's Theorem *Let $p(x)$ be a polynomial of degree 4 such that the curve $y = p(x)$ has two distinct points of inflection, A and B. Suppose the straight line passing through A and B intersects the curve again at points P and Q, where the x-coordinates of P, A, B and Q are $p < a < b < q$, respectively. Then*

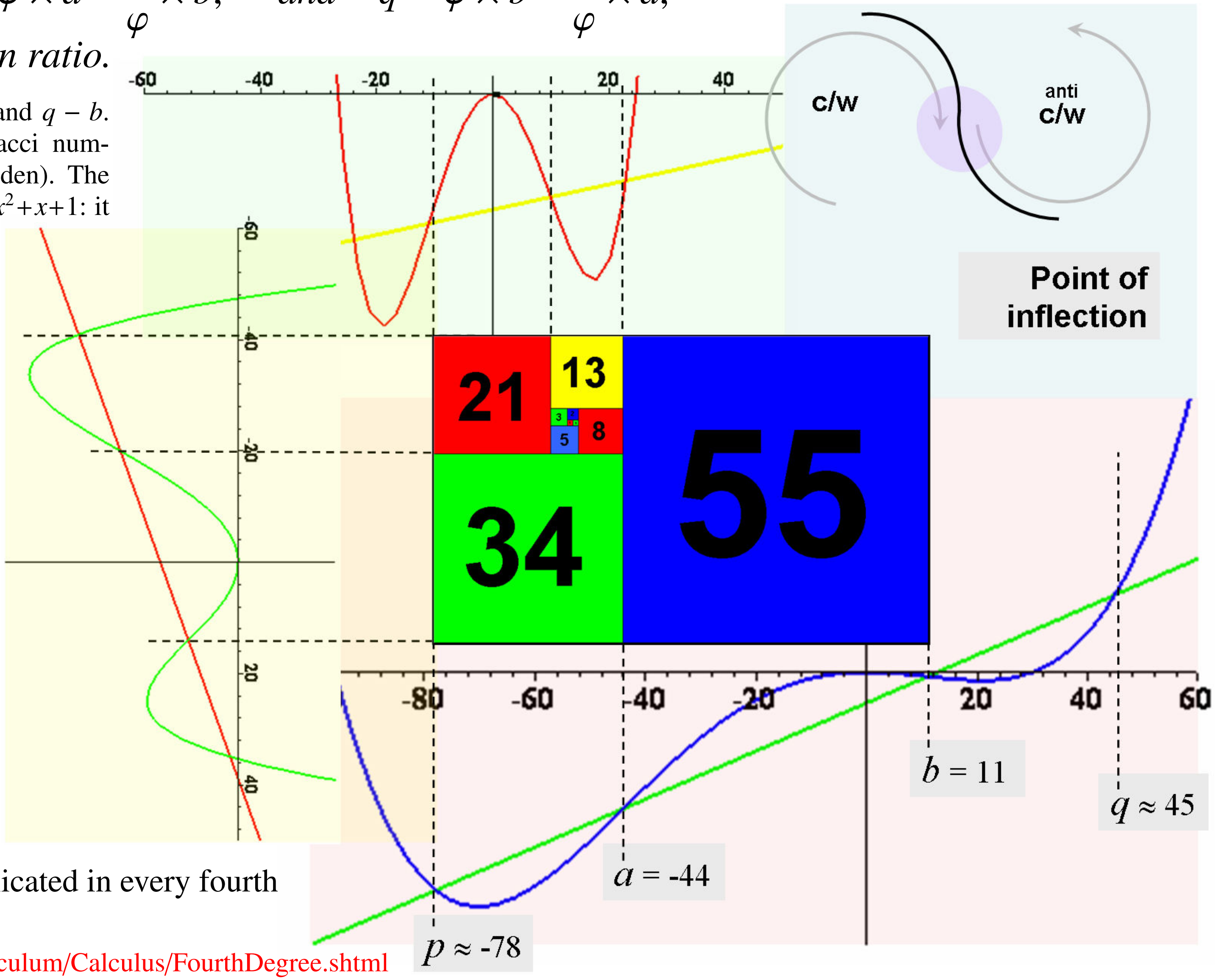
$$p = \varphi \times a - \frac{1}{\varphi} \times b, \quad \text{and} \quad q = \varphi \times b - \frac{1}{\varphi} \times a,$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio.

As a corollary, $b - a$ is in the golden ratio to $a - p$ and $q - b$. This is illustrated here with various pairs of Fibonacci numbers (of course, their ratios are only *approximately* golden). The curve at the bottom, for example, is $y = x^4+66x^3-2904x^2+x+1$: it has points of inflection at $x = -44$ and $x = 11$, and the straight line joining them intersects the curve again at $x \approx -78$. These curves are constructed as solutions to the differential equation

$$\frac{d^2y}{dx^2} = 12k(x-a)(x-b),$$

using the fact that the second derivative has zeros at points of inflection.



This surprising property of quartics was discovered in 2004 by US high school teacher Lin McMullin whose clever idea of twice integrating a factorised quadratic led naturally to the appearance of the golden ratio. It remains true, provided the straight line equation is specified in a suitably general manner, even when a and b, or the constants of integration which are introduced, are not distinct or are complex: φ is unavoidably implicated in every fourth degree equation.

Web link: www.cut-the-knot.org/Curriculum/Calculus/FourthDegree.shtml

Further reading: *The Divine Proportion: A Study in Mathematical Beauty* by H.E. Huntley, Dover Publications, 1970.