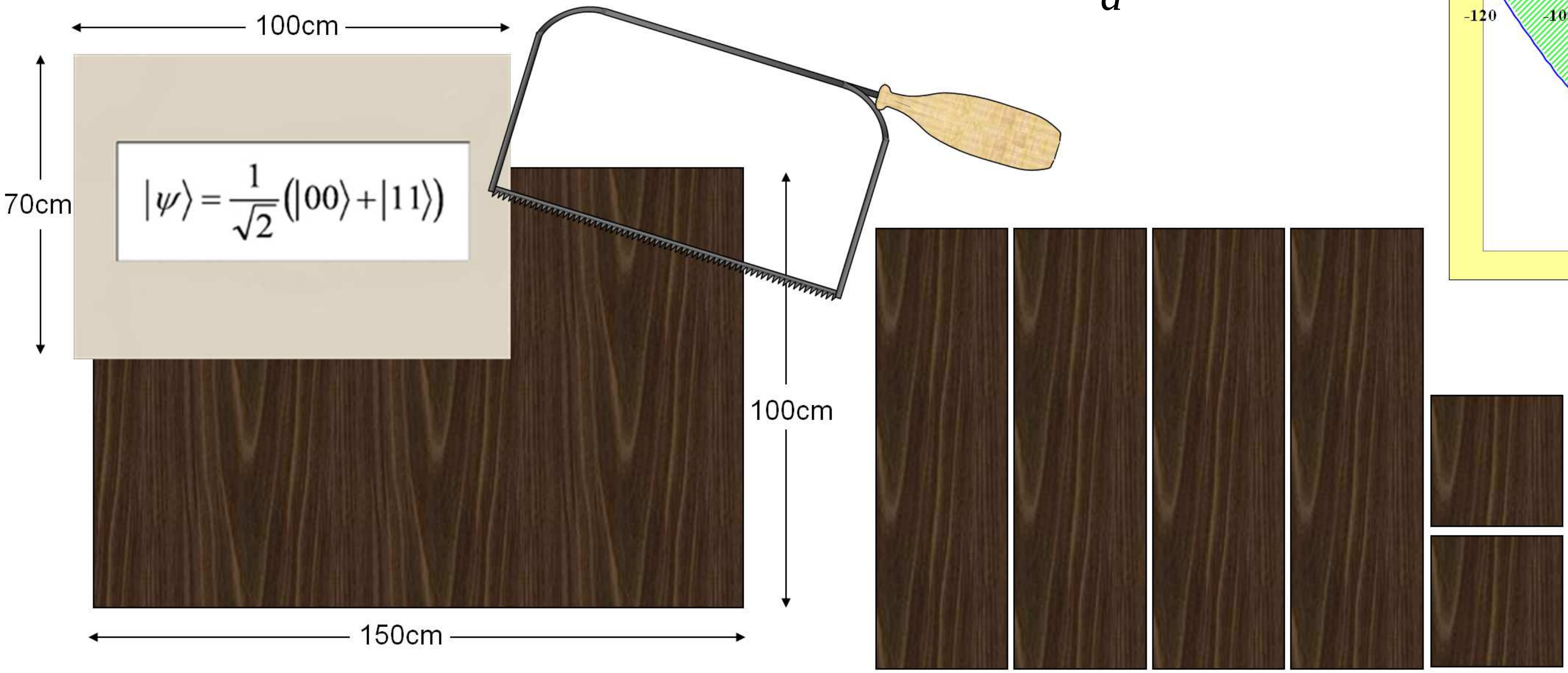
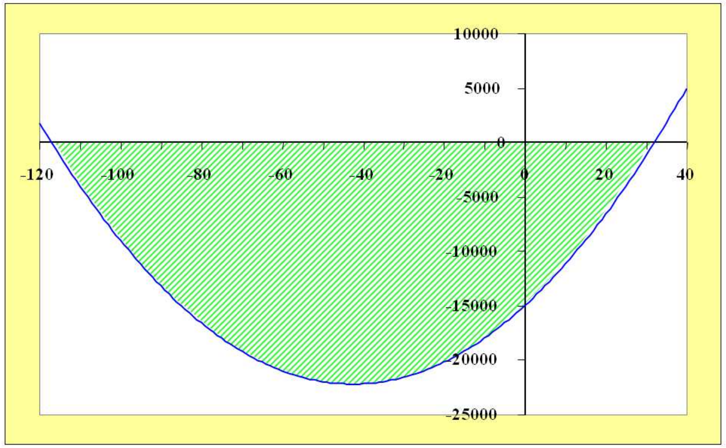
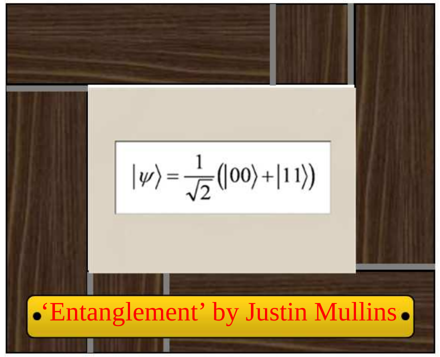
The Quadratic Formula *The quadratic equation* $ax^2 + 2bx + c = 0$*, with complex values of* a*,* b *and* c*, is solved by the two, possibly equal,* x *values*

$$x = \frac{-b \pm \sqrt{b^2 - ac}}{a}.$$







The problem shown above is one of many ways in which quadratic equations may arise: choose a width in such a way as to best fill a given area. In this case, the width of a picture frame for a picture 70cm × 100cm is to be chosen so as to best use up a 100cm × 150cm sheet of walnut. If the frame has width x cm, then its area is $2 \times 100 \times x$ cm^2 (for the top and bottom) + $2 \times 70 \times x$ cm^2 (for the sides) + $4 \times x^2$ cm^2 (for the corners). So we must chose x so that $4x^2 + 340x \leq 100 \times 150$ or, rearranging, $4x^2 + 340x - 15000 \leq 0$. The graph, top-right, plots the curve $y = 4x^2 + 340x - 15000$; any x value within the green shaded part solves the inequality, with the extreme values being the solutions given by the quadratic formula, although only the positive value ≈32.04 is of interest to us. I have chosen to cut the sheet of walnut into strips of width 30cm, wasting 1200cm^2 and giving the framing shown above, bottom-right. A width of 32cm is obviously possible and would waste only 24cm^2 but I could find no way of cutting up the sheet that was not unduly tortuous (perhaps you can do better?)

The scenario shown here would have been familiar to the ancients and was readily solved in Egyptian times (say, 1500BC). The quadratic formula as we know it today (here adopting a formulation suggested by Rob J. Low) is due to the 12th century Indian mathematician Bhaskara.

Proof (by completing the square, beautifully visualised at www.jensilvermath.com/lessons/algebra/)

$$\begin{aligned} & ax^2 + 2bx + c = 0 \\ \Leftrightarrow\ & a\left(x + \frac{b}{a}\right)^2 - \frac{b^2}{a} + c = 0 \\ \Leftrightarrow\ & \left(x + \frac{b}{a}\right)^2 = \frac{b^2 - ac}{a^2} \\ \Leftrightarrow\ & x = -\frac{b}{a} \pm \sqrt{\frac{b^2 - ac}{a^2}} \\ & \quad = \frac{-b \pm \sqrt{b^2 - ac}}{a}. \end{aligned}$$

Web link: plus.maths.org/issue30/features/quadratic/index-gifd.html

Further reading: *Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook* by Victor Katz (ed.), Princeton University Press, 2007.