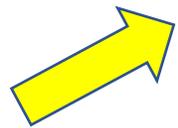
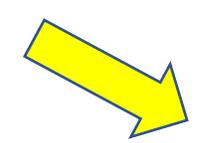


$P - 1$ as a lower triangular matrix

1										
1	1									
1	2	1								
1	3	3	1							
1	4	6	4	1						
1	5	10	10	5	1					
1	6	15	20	15	6	1				
1	7	21	35	35	21	7	1			
1	8	28	56	70	56	28	8	1		
1	9	36	84	126	126	84	36	9	1	
1	10	45	120	210	252	210	120	45	10	1
...										



	1	2	3	4	5	6	...
0	0						
1	0	0					
2	0	1	0				
3	0	2	2	0			
4	0	3	5	3	0		
5	0	4	9	9	4	0	
6	0	5	14	19	14	5	0
⋮							



1	0	0	0	0
2	2	0	0	0
3	5	3	0	0
4	9	9	4	0
5	14	19	14	5

A014430: Pascal – 1

The OEIS is supported by [the many generous donors to the OEIS Foundation](#).



founded in 1964 by N. J. A. Sloane

[Hints](#)

(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

A014430 Subtract 1 from Pascal's triangle, read by rows. 9

1, 2, 2, 3, 5, 3, 4, 9, 9, 4, 5, 14, 19, 14, 5, 6, 20, 34, 34, 20, 6, 7, 27, 55, 69, 55, 27, 7, 8, 35, 83, 125, 125, 83, 35, 8, 9, 44, 119, 209, 251, 209, 119, 44, 9, 10, 54, 164, 329, 461, 461, 329, 164, 54, 10, 11, 65, 219, 494, 791, 923, 791, 494, 219, 65, 11 ([list](#); [table](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0,2

COMMENTS Each value of the sequence $(T(x,y))$ is equal to the sum of all values in Pascal's Triangle that are in the rectangle defined by the tip $(0,0)$ and the position (x,y) .
- Florian Kleedorfer (florian.kleedorfer(AT)austria.fm), May 23 2005

To clarify [A014430](#) and [A129696](#): We subtract $I =$ Identity matrix from Pascal's triangle to obtain the beheaded variant, [A074909](#). Then take column sums starting from the top of [A074909](#) to get triangle [A014430](#). Row sums of the inverse of triangle [A014430](#) gives the Bernoulli numbers, [A027641/A026642](#). Alternatively, triangle [A014430](#) as an infinite lower triangular matrix * [the Bernoulli numbers as a vector] = $[1, 1, 1, \dots]$. Given the B_n version starting $(1, 1/2, 1/6, \dots)$ triangle [A014430](#) * the B_n vector $[1, 1/2, 1/6, 0, -1/30, \dots]$ = the triangular numbers. - Gary W. Adamson, Mar 13 2012

If regarded as a symmetric array of the form

```
1 2 3 4 5 ...
2 5 9 14 20 ...
3 9 19 34 55 ...
4 14 34 69 125 ...
5 20 55 125 251 ...
6 27 83 209 461 ...
7 35 119 329 791 ...
8 44 164 494 1286 ...
9 54 219 714 2001 ...
```

it contains the rows (and columns) [A000096](#), [A062748](#), [A063258](#), [A062988](#), [A124089](#), ..., [A035927](#) and so on and counts the multisets of digits of numbers in base $b \geq 2$ with $d \geq 1$ digits (equivalent to the comment in [A035927](#)). - R. J. Mathar, Apr 25 2016

OEIS sequence A014430 Part 1

If regarded as a symmetric array of the form

1	2	3	4	5	...
2	5	9	14	20	...
3	9	19	34	55	...
4	14	34	69	125	...
5	20	55	125	251	...
6	27	83	209	461	...
7	35	119	329	791	...
8	44	164	494	1286	...
9	54	219	714	2001	...

1

2, 2

3, 5, 3

4, 9, 9, 4

5, 14, 19, 14, 5

6, 20, 34, 34, 20, 6

7, 27, 55, 69, 55, 27, 7

8, 35, 83, 125, 125, 83, 35, 8

9, 44, 119, 209, 251, 209, 119, 44, 9

10, 54, 164, 329, 461, 461, 329, 164, 54, 10

11, 65, 219, 494, 791, 923, 791, 494, 219, 65, 11

12, 77, 285, 714, 1286, 1715, 1715, 1286, 714, 285, 77, 12

it contains the rows (and columns) [A000096](#), [A062748](#), [A063258](#), [A062988](#), [A124089](#), ..., [A035927](#) and so on and counts the multisets of digits of numbers in base $b \geq 2$ with $d \geq 1$ digits (equivalent to the comment in [A035927](#)). - [R. J. Mathar](#), Apr 25 2016

16, 135, 679, 2379, 6187, 12375, 19447, 24309, 24309, 19447, 12375, 6187, 2379, 679, 135, 16

17, 152, 815, 3059, 8567, 18563, 31823, 43757, 48619, 43757, 31823, 18563, 8567, 3059, 815, 152, 17

18, 170, 968, 3875, 11627, 27131, 50387, 75581, 92377, 92377, 75581, 50387, 27131, 11627, 3875, 968, 170, 18

19, 189, 1139, 4844, 15503, 38759, 77519, 125969, 167959, 184755, 167959, 125969, 77519, 38759, 15503, 4844, 1139, 189, 19

20, 209, 1329, 5984, 20348, 54263, 116279, 203489, 293929, 352715, 352715, 293929, 203489, 116279, 54263, 20348, 5984, 1329, 209

21, 230, 1539, 7314, 26333, 74612, 170543, 319769, 497419, 646645, 705431, 646645, 497419, 319769, 170543, 74612, 26333, 7314, 1539

Multisets of base $b \geq 2$ numbers with $d \geq 1$ digits

		d						
		1	2	3	4	5	6	...
b	2	1	2	3	4	5	6	
	3	2	5	9	14			
	4	3	9	19				
	5	4	14					
	6	5						
	5	6						
	⋮							

$b = 3, d = 3$

1	0	0
2	0	0
1	1	0
1	2	0
2	2	0
1	1	1
1	2	1
1	2	2
2	2	2

$b = 4, d = 2$

1	0
2	0
3	0
1	1
1	2
1	3
2	2
2	3
3	3

Pascal and the Christmas stocking identity

	0	1	2	3	4	5	6	7	8	...
0	1									
1	1	1								
2	1	2	1							
3	1	3	3	1						
4	1	4	6	4	1					
5	1	5	10	10	5	1				
6	1	6	15	20	15	6	1			
7	1	7	21	35	35	21	7	1		
8	1	8	28	56	70	56	28	8	1	
⋮										

$$\binom{n+1}{m+1} = \sum_{r=m}^n \binom{r}{m}$$

OEIS sequence A014430 Part 2

COMMENTS

Each value of the sequence $(T(x,y))$ is equal to the sum of all values in Pascal's Triangle that are in the rectangle defined by the tip $(0,0)$ and the position (x,y) .

- Florian Kleedorfer (florian.kleedorfer(AT)austria.fm), May 23 2005

Q	0	1	2	3	4	5	...
0	1						
1	2	2					
2	3	5	3				
3	4	9	9	4			
4	5	14	19	14	5		
5	6	20	34	34	20	6	
⋮							

P	0	1	2	3	4	5	...
0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
⋮							

Kleedorfer's formula (?)

Proof of Florian Kleedorfer's formula: Take sums of the columns of the rectangle - these are all binomial coefficients by the Hockey Stick Identity. Note the locations of these coefficients: They form a row going almost all the way to the edge, only missing the 1 - apply the Hockey Stick Identity again. - [James East](#), Jul

Q	0	1	2	3	4	5	...
0	1						
1	2	2					
2	3	5	3				
3	4	9	9	4			
4	5	14	19	14	5		
5	6	20	34	34	20	6	
⋮							

P	0	1	2	3	4	5	...
0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
⋮							

Inverting (infinite) lower triangular matrices

$$(a)^{-1} = \begin{pmatrix} 1 \\ a \end{pmatrix}$$

$$\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}^{-1} = \frac{1}{ac} \begin{pmatrix} c & 0 \\ -b & a \end{pmatrix} = \begin{pmatrix} \frac{1}{a} & 0 \\ -\frac{b}{ac} & \frac{1}{c} \end{pmatrix}$$

$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}^{-1} = \frac{1}{acf} \begin{pmatrix} cf & 0 & 0 \\ -bf & af & 0 \\ be - cd & -ae & ac \end{pmatrix} = \begin{pmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{b}{ac} & \frac{1}{c} & 0 \\ \frac{be - cd}{acf} & -\frac{e}{cf} & \frac{1}{f} \end{pmatrix}$$

$$X[[n]||[n]]^{-1} = X^{-1}[[n]||[n]]$$
$$[n] = \{1, \dots, n\}$$

Inverting Pascal

$$\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ -1 & 3 & -3 & 1 & 0 \\ 1 & -4 & 6 & -4 & 1 \end{bmatrix}$$

$$P_{ij}^{-1} = (-1)^{i+j} P_{ij}$$

Inverting Pascal minus 1

$$[1], \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 5 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 5 & 3 & 0 \\ 4 & 9 & 9 & 4 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 \\ 4 & 9 & 9 & 4 & 0 \\ 5 & 14 & 19 & 14 & 5 \end{bmatrix}$$

$$[1], \begin{bmatrix} 1 & 0 \\ -1 & \frac{1}{2} \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ \frac{2}{3} & -\frac{5}{6} & \frac{1}{3} \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & \frac{1}{2} & 0 & 0 \\ \frac{2}{3} & -\frac{5}{6} & \frac{1}{3} & 0 \\ -\frac{1}{4} & \frac{3}{4} & -\frac{3}{4} & \frac{1}{4} \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{2}{3} & -\frac{5}{6} & \frac{1}{3} & 0 & 0 \\ -\frac{1}{4} & \frac{3}{4} & -\frac{3}{4} & \frac{1}{4} & 0 \\ -\frac{1}{30} & -\frac{1}{3} & \frac{5}{6} & -\frac{7}{10} & \frac{1}{5} \end{bmatrix}$$

$$Q_{ij}^{-1} = ?$$

Inverting Pascal minus 1 and summing rows

$$\left[\begin{array}{cccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ \frac{2}{3} & -\frac{5}{6} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{30} & -\frac{1}{3} & \frac{5}{6} & -\frac{7}{10} & \frac{1}{5} & 0 & 0 & 0 & -\frac{1}{30} \\ \frac{1}{12} & -\frac{1}{12} & -\frac{5}{12} & \frac{11}{12} & -\frac{2}{3} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{42} & \frac{1}{6} & -\frac{1}{6} & -\frac{1}{2} & 1 & -\frac{9}{14} & \frac{1}{7} & 0 & \frac{1}{42} \\ -\frac{1}{12} & \frac{1}{12} & \frac{7}{24} & -\frac{7}{24} & -\frac{7}{12} & \frac{13}{12} & -\frac{5}{8} & \frac{1}{8} & 0 \end{array} \right]$$

$$\sum_j Q_{ij}^{-1} = B_i$$

The Bernoulli numbers $B_0 = 1, B_1 = -\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0, \dots$

THEOREM OF THE DAY

Faulhaber's Formula *The sum of the r -th powers of the first n positive integers is given by*

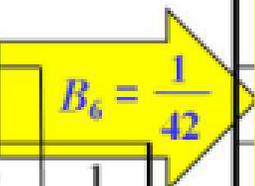
$$1^r + 2^r + \dots + n^r = \frac{1}{r+1} \sum_{k=0}^r (-1)^k \binom{r+1}{k} B_k n^{r-k+1}.$$

The calculation of our sum of r -th powers involves a double scan of the $(r+1)$ -th row of Pascal's triangle. We need to produce the first $r+1$ so-called **Bernoulli numbers**, denoted by B_0, B_1, \dots, B_r . Suppose that we have B_0, B_1, \dots, B_{r-1} , then we can extract B_r by solving the equation $\sum_{i=0}^r \binom{r+1}{i} B_i = 0$. In the example below, $r = 6$; the values of B_0, \dots, B_5 are shown ($B_0 = 1, B_1 = -1/2$, etc) and the equation yields the value $B_6 = 1/42$. (The properties of Pascal's triangle conspire to give every Bernoulli number of odd index, beyond B_1 , the value zero.)

$$\begin{aligned} &1^6 + 2^6 + 3^6 + 4^6 + 5^6 + \dots + n^6 \\ &= \frac{1}{7} \left(n^7 + \frac{7}{2} n^6 + \frac{21}{6} n^5 - \frac{35}{30} n^3 + \frac{7}{42} n \right) \\ &= \frac{1}{42} (6n^7 + 21n^6 + 21n^5 - 7n^3 + n) \\ &= \frac{1}{42} n(n+1)(2n+1)(3n^2 + 6n^3 - 3n + 1) \end{aligned}$$

1										
1	1									
1	2	1								
1	3	3	1							
1	4	6	4	1						
1	5	10	10	5	1					
+	6	15	20	15	6	+				
1	7	21	35	35	21	7	1			
B_0	B_1	B_2	B_3	B_4	B_5	B_6	8	1		
n^7	n^6	n^5	n^4	n^3	n^2	n^1	36	9	1	
1	10	45	120	210	252	210	120	45	10	1

1	5	10	10	5	1					
1	6	15	20	15	6	1				
1	7	21	35	35	21	7	1			
1	$-\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{30}$	0	B_6	8	1		
1	9	36	84	126	126	84	36	9	1	
			...							



Seki Takakazu and Bernoulli

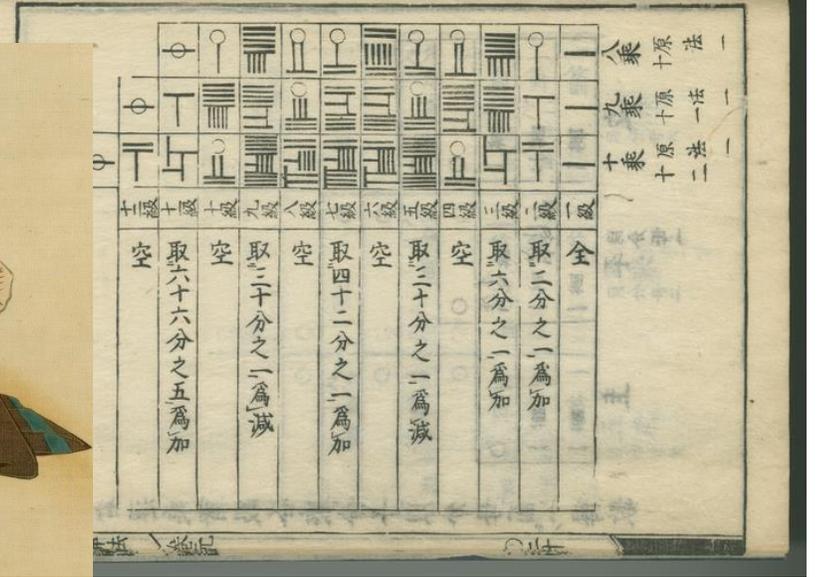
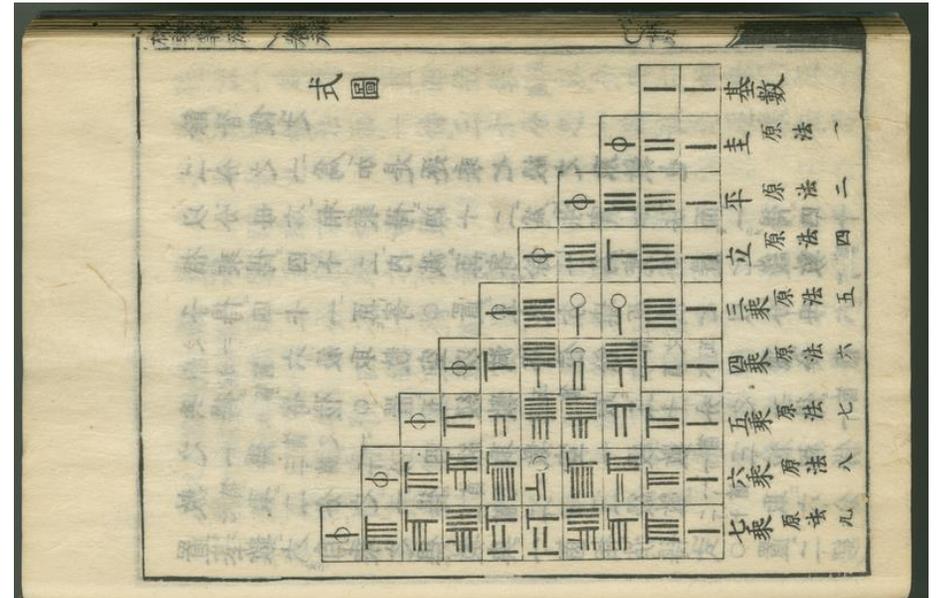
$$\begin{aligned}
 f n &= \frac{1}{2} n n + \frac{1}{2} n \\
 f n n &= \frac{1}{3} n^3 + \frac{1}{2} n n + \frac{1}{6} n \\
 f n^3 &= \frac{1}{4} n^4 + \frac{1}{2} n^3 + \frac{1}{4} n n \\
 f n^4 &= \frac{1}{5} n^5 + \frac{1}{2} n^4 + \frac{1}{3} n^3 - \frac{1}{30} n \\
 f n^5 &= \frac{1}{6} n^6 + \frac{1}{2} n^5 + \frac{5}{12} n^4 - \frac{1}{12} n n \\
 f n^6 &= \frac{1}{7} n^7 + \frac{1}{2} n^6 + \frac{1}{2} n^5 - \frac{1}{6} n^3 + \frac{1}{42} n \\
 f n^7 &= \frac{1}{8} n^8 + \frac{1}{2} n^7 + \frac{7}{12} n^6 - \frac{7}{24} n^4 + \frac{1}{12} n n \\
 f n^8 &= \frac{1}{9} n^9 + \frac{1}{2} n^8 + \frac{2}{3} n^7 - \frac{7}{15} n^5 + \frac{2}{9} n^3 - \frac{1}{30} n \\
 f n^9 &= \frac{1}{10} n^{10} + \frac{1}{2} n^9 + \frac{3}{4} n^8 - \frac{7}{10} n^6 + \frac{1}{2} n^4 - \frac{1}{12} n n \\
 f n^{10} &= \frac{1}{11} n^{11} + \frac{1}{2} n^{10} + \frac{5}{6} n^9 - n^7 + n^5 - \frac{1}{2} n^3 + \frac{5}{66} n
 \end{aligned}$$

Quin imò qui legem progressionis inibi attentius enspexit, eum
 am continuare poterit absque his ratiociniis ambabimus: Sum
 c pro potestatis cujuslibet exponente, fit summa omnium n^c se

$$\begin{aligned}
 \int n^c &= \frac{1}{c+1} n^{c+1} + \frac{1}{2} n^c + \frac{c}{2} A n^{c-1} + \frac{c \cdot c - 1 \cdot c - 2}{2 \cdot 3 \cdot 4} B n^{c-2} \\
 &+ \frac{c \cdot c - 1 \cdot c - 2 \cdot c - 3 \cdot c - 4}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} C n^{c-3} \\
 &+ \frac{c \cdot c - 1 \cdot c - 2 \cdot c - 3 \cdot c - 4 \cdot c - 5 \cdot c - 6}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} D n^{c-4} \dots \& \text{ita deinceps,}
 \end{aligned}$$

exponentem potestatis ipsius n continuè minuendo binario, quosque per
 veniatur ad n vel nn . Literae capitales A, B, C, D & c. ordine denotant
 coëfficientes ultimorum terminorum pro $f n n$, $f n^4$, $f n^6$, $f n^8$, & c.
 nempe

$$A = \frac{1}{6}, B = -\frac{1}{30}, C = \frac{1}{42}, D = -\frac{1}{30}.$$



Jakob Bernoulli's *Ars Conjectandi* (1713)

Seki Takakazu's *Katsuyō Sanpō* (1712)

Ramanujan and odd zeta

The zeta function is

$$\xi(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \dots$$

Euler (1739) discovered a closed form expression for $\xi(s)$ for even integers s in terms of the Bernoulli numbers.

Ramanujan discovered an infinite series for odd values $s = 2n + 1$. In the case where n is odd this specialises to the following (Mathias Lerch, 1901):

$$\xi(2n + 1) = \frac{1}{2} \tau^{2n+1} \sum_{k=0}^{n+1} (-1)^{k+1} \frac{B_{2k}}{(2k)!} \frac{B_{2n+2-2k}}{(2n+2-2k)!} - 2 \sum_{k=1}^{\infty} \frac{k^{-2n-1}}{e^{k\tau} - 1}$$



Srinivasa Ramanujan

(Note for large odd n the final term goes to zero, leaving a rational multiple of a power of τ).

Faulhaber's formula via Pascal – 1

$$1^r + 2^r + 3^r + \dots + n^r = U(n, r)Q_r^{-1}\mathbf{1}$$

where

$$U(n, r) = \frac{1}{r+1} \left(\binom{r+1}{0} n^{r+1}, \dots, (-1)^k \binom{r+1}{k} n^{r+1-k}, \dots, (-1)^r \binom{r+1}{r} n^1 \right)$$

and

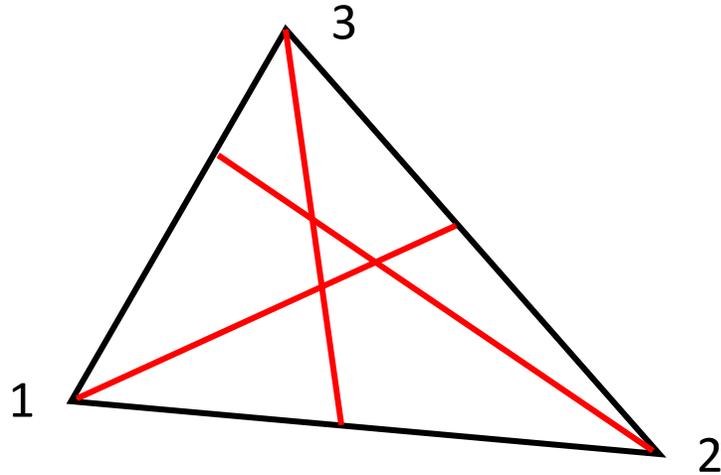
$$\mathbf{1} = (1, \dots, 1).$$

$$\begin{aligned} 1^3 + 2^3 + 3^3 + 4^3 + 5^3 &= U(5, 3)Q_3^{-1}\mathbf{1} \\ &= \frac{1}{4} \left(\binom{4}{0} 5^4, -\binom{4}{1} 5^3, \binom{4}{2} 5^2, -\binom{4}{3} 5^1 \right) \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1/2 & 0 & 0 \\ 2/3 & -5/6 & 1/3 & 0 \\ -1/4 & 3/4 & -3/4 & 1/4 \end{pmatrix} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \\ &= \left(\frac{625}{4}, -125, \frac{75}{2}, -5 \right) \times \left(1, -\frac{1}{2}, \frac{1}{6}, 0 \right)^{-1} = 225. \end{aligned}$$

OEIS sequence A014430 Part 4: Intersecting chords in convex polygons

For an n -vertex convex polygon we are interested in enumerating configurations of n pairwise intersecting chords, each joining a distinct vertex to a non-adjacent edge.

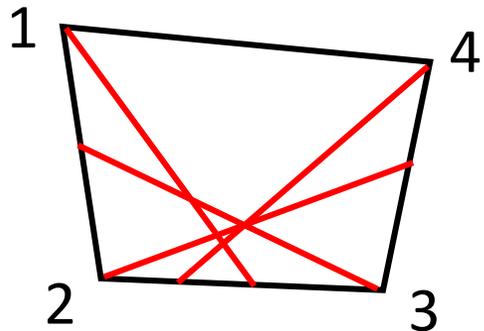
$n = 3$



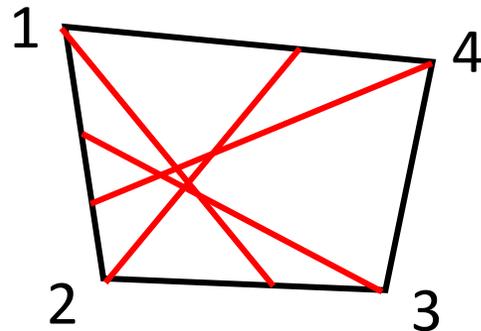
1, 2
2, 3
3, 1

We take the vertices in anti-clockwise order and denote a chord from vertex u to edge vw by the ordered pair (u, v) .

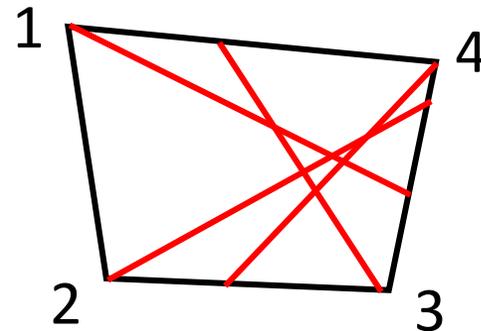
$n = 4$



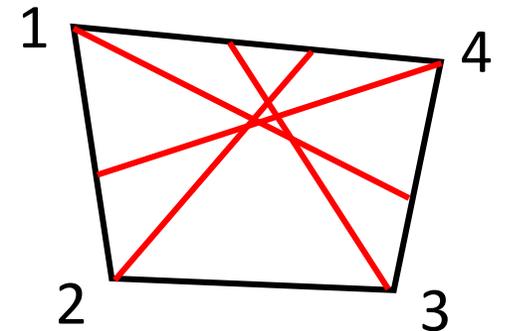
1, 2
2, 3
3, 1
4, 2



1, 2
2, 4
3, 1
4, 1

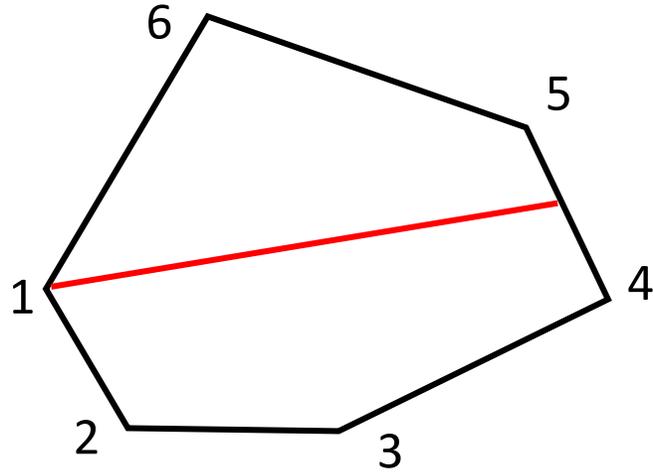


1, 3
2, 3
3, 4
4, 2



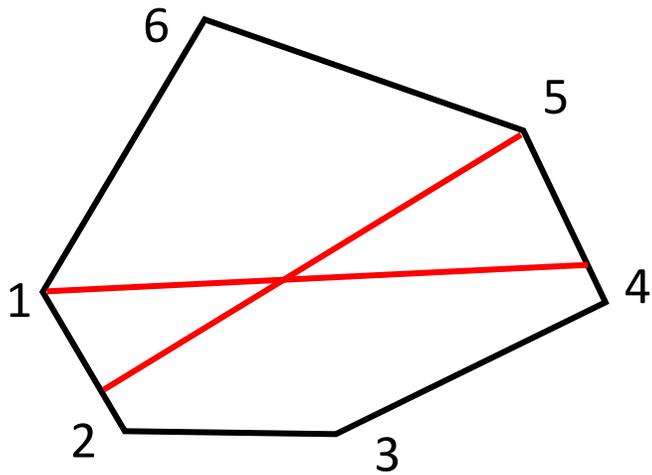
1, 3
2, 4
3, 4
4, 1

Another way of looking at it

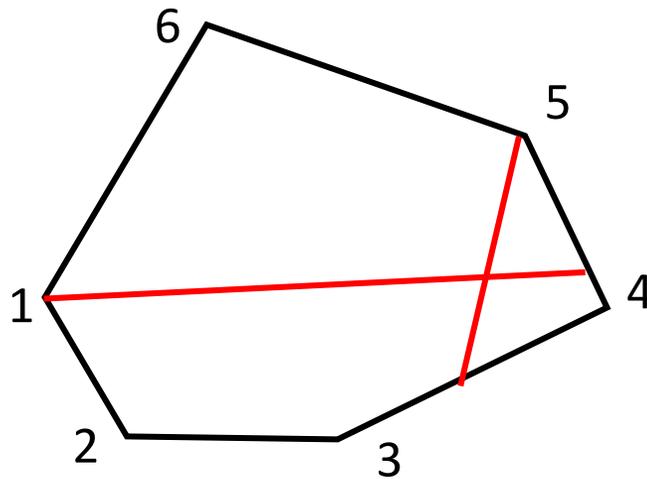


Suppose we have placed a chord, e.g. 1 4, as shown left.

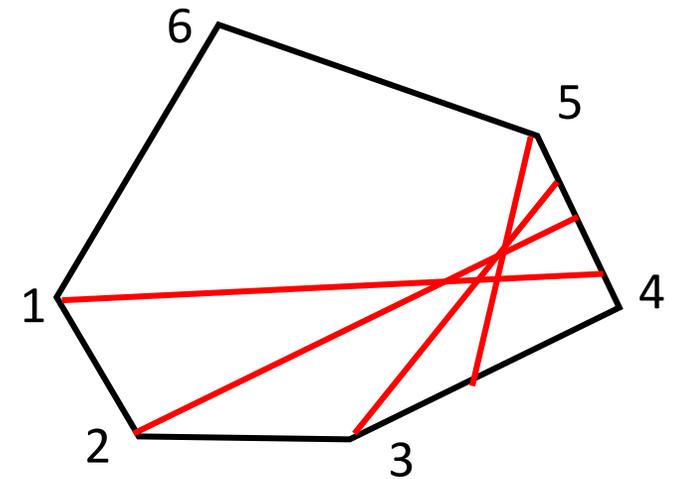
Consider the possibilities for vertex 5.



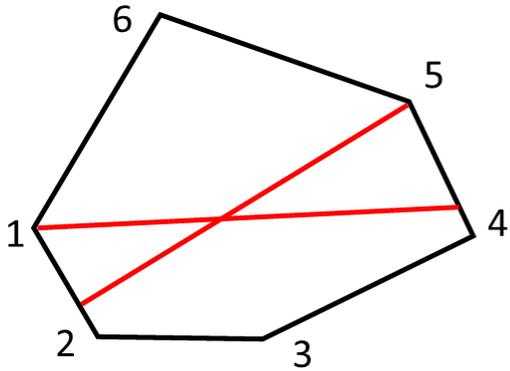
Continuing 5 1 is legitimate



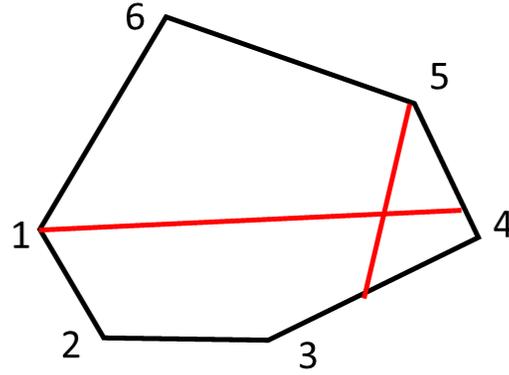
Continuing 5 3 is fine but only if 2 and 3 go to edge 4



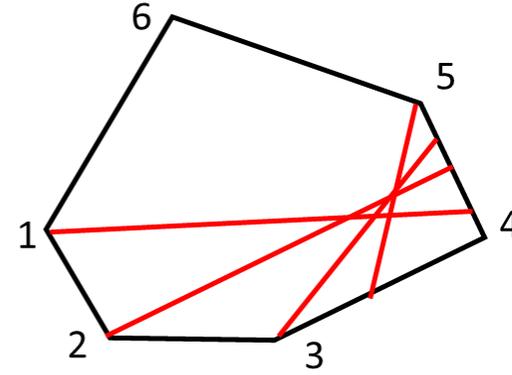
Another way of looking at it



Continuing 5 1 is legitimate



Continuing 5 3 is fine ... but only if 2 and 3 go to edge 4



Representing this in tabular form.

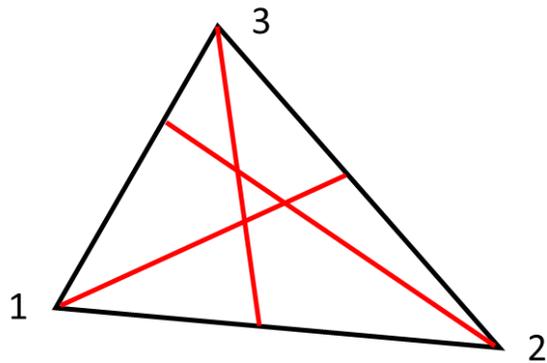
	1	2	3	4	5	6
1				1		
2						
3						
4						
5	1					
6						

	1	2	3	4	5	6
1				1		
2				1		
3				1		
4						
5			1			
6						

A chord entry that does not have an entry directly below it must have an entry diagonally opposite the cell to its right.

Enumerating $n = 3$ and $n = 4$ in tabular form

$n = 3$

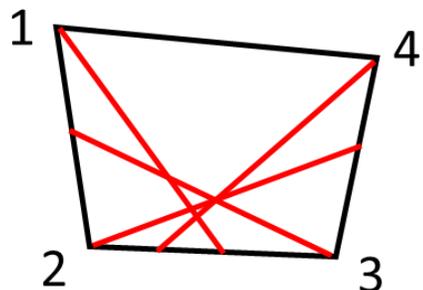


1, 2
2, 3
3, 1

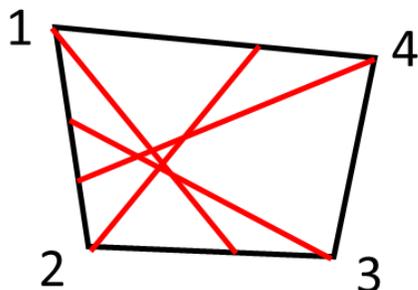
	1	2	3
1			1
2			1
3	1		

Note wrap-around
from entry 2 3

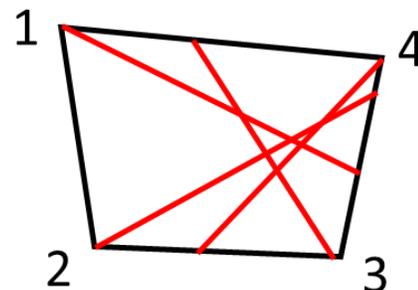
$n = 4$



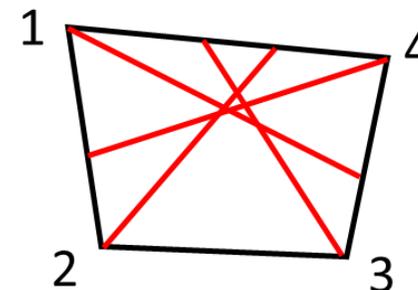
1, 2
2, 3
3, 1
4, 2



1, 2
2, 4
3, 1
4, 1



1, 3
2, 3
3, 4
4, 2



1, 3
2, 4
3, 4
4, 1

	1	2	3	4
1		1		
2			1	
3	1			
4		1		

	1	2	3	4
1		1		
2				1
3	1			
4	1			

	1	2	3	4
1			1	
2			1	
3				1
4		1		

	1	2	3	4
1			1	
2				1
3				1
4	1			

Enumerating by edge targeted from vertex 1 (table entry in row 1)

Enumerated as tables by hand

	1,2	1,3	1,4	1,5	1,6	Totals
3	1					1
4	2	2				4
5	3	5	3			11
6	4	9	9	4		26
7	5	14	19	14	5	57

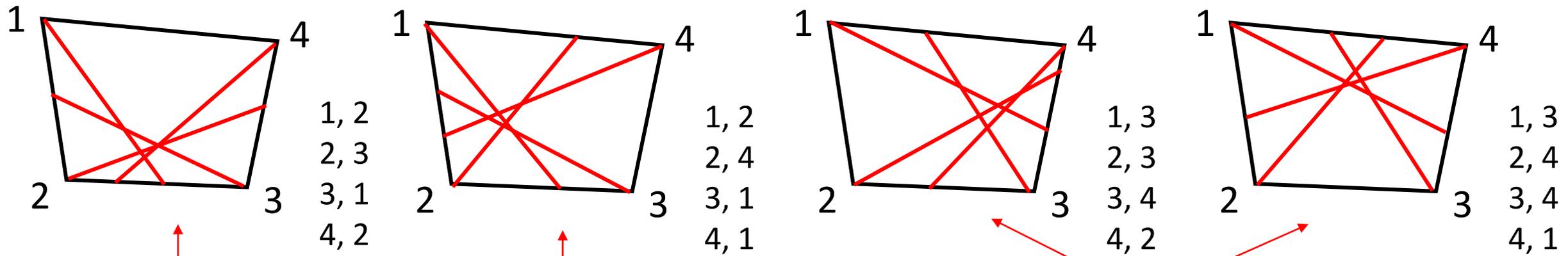


A000295 in OEIS = 2nd column of Euler's triangle

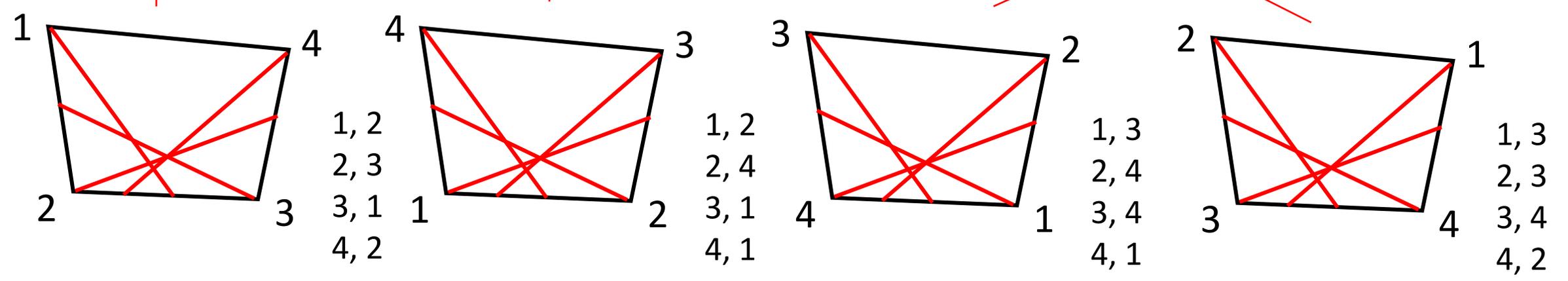
How can we interpret the symmetry in the table?

Under rotational symmetry

$n = 4$



Rotating the first configuration:



So up to rotational symmetry, there is only one configuration for $n = 4$

Classifying by column sum

	1	2	3	4
1		1		
2			1	
3	1			
4		1		

1 2 1 0

	1	2	3	4
1		1		
2				1
3	1			
4	1			

2 1 0 1

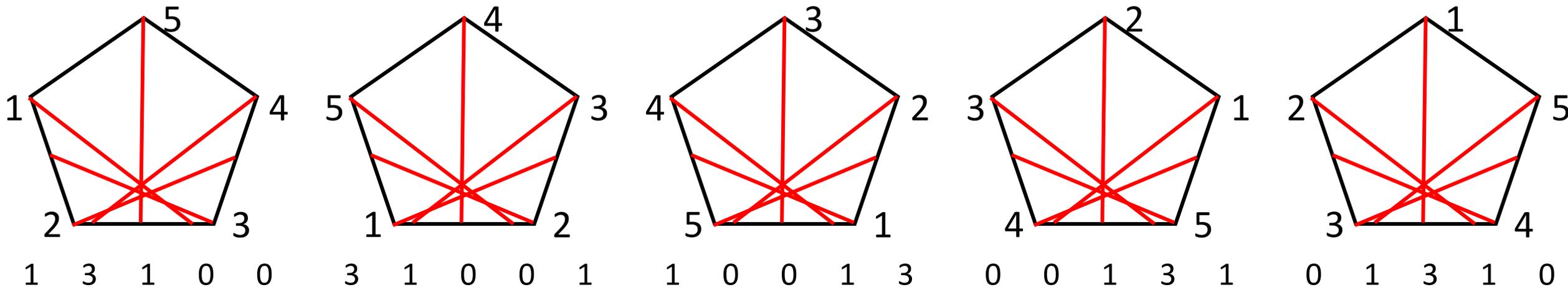
	1	2	3	4
1			1	
2			1	
3				1
4		1		

0 1 2 1

	1	2	3	4
1			1	
2				1
3				1
4	1			

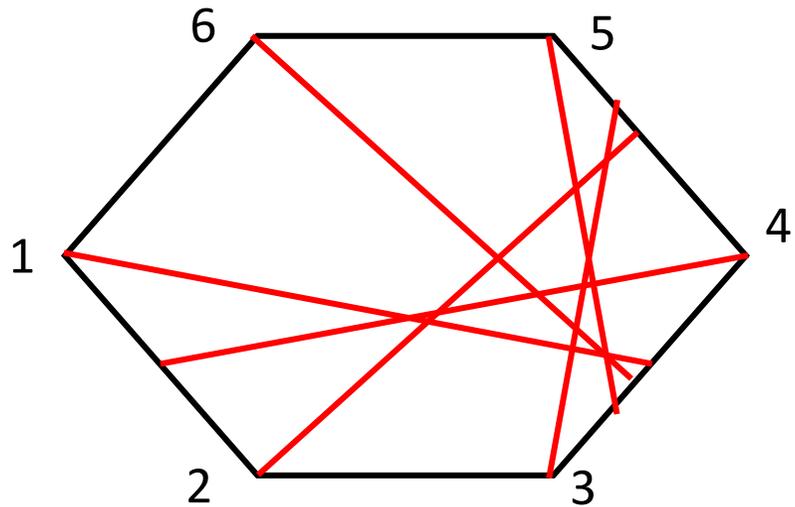
1 0 1 2

For $n = 4$ we may enumerate configurations as one column-sum vector rotated round



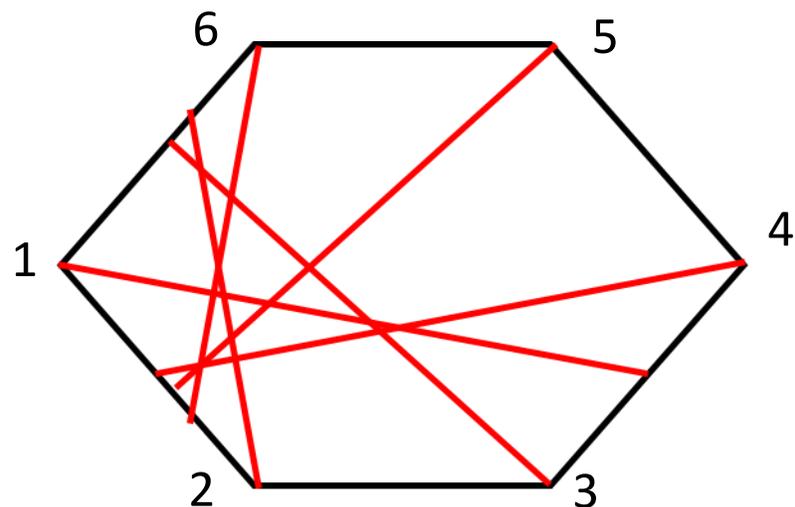
Works for $n = 5$ as well. There are 11 configurations deriving from rotations of 3 asymmetrical.

Non-rotational symmetry $n = 6$



	1	2	3	4	5	6
1			1			
2				1		
3				1		
4	1					
5			1			
6			1			

1 0 3 2 0 0



	1	2	3	4	5	6
1			1			
2						1
3						1
4	1					
5	1					
6	1					

3 0 1 0 0 2

Configurations up to rotational symmetry $n = 3, \dots, 7$

3	4	5	6	7
1	1	3	5	9

0 1 3 6 2 7
: 13
: 20
23 **OEIS** 12
10 22 11 21

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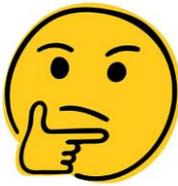





[Other ways](#)

[Hints](#)

(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))



Search: seq:1,1,3,~~5~~,9

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[A000213](#) Tribonacci numbers: $a(n) = a(n-1) + a(n-2) + a(n-3)$ with $a(0)=a(1)=a(2)=1$. +30
143

(Formerly M2454 N0975)

1 1 1 3 5 9 17 31 57 105 193 355 653 1201 2209 4063 7473 13745 25281 46499