Pascal's Rule *For any positive integers n and k,*

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$

This is read as "$n + 1$ choose $k = n$ choose $k + n$ choose $k - 1$", invoking its combinatorial interpretation: the number of choices if you must select k objects from $n + 1$ is the same as the number of choices if you are selecting from n objects and have an initial choice of whether to take k or $k - 1$. This suggests an easy **combinatorial proof:** a choice of k from a row of $n + 1$ objects will either exclude the last object, in which case all k must be chosen from the first n; or it will include it, leaving $k - 1$ to be chosen from the first n. QED

An algebraic proof:

$$\binom{n}{k} + \binom{n}{k-1} = \frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-(k-1))!}$$

$$= \frac{(n-k+1)\times n! + k \times n!}{k!(n-k+1)!}$$

$$= \frac{n!(n-k+1+k)}{k!(n+1-k)!}$$

$$= \frac{(n+1)!}{k!((n+1)-k)!} = \binom{n+1}{k}. \quad \text{QED}$$

Pascal's Triangle
Rows are numbered from zero; cells in each row are likewise numbered from zero. Row zero consists of $\binom{0}{0} = 1$; the n-th row starts with $\binom{n}{0} = 1$.
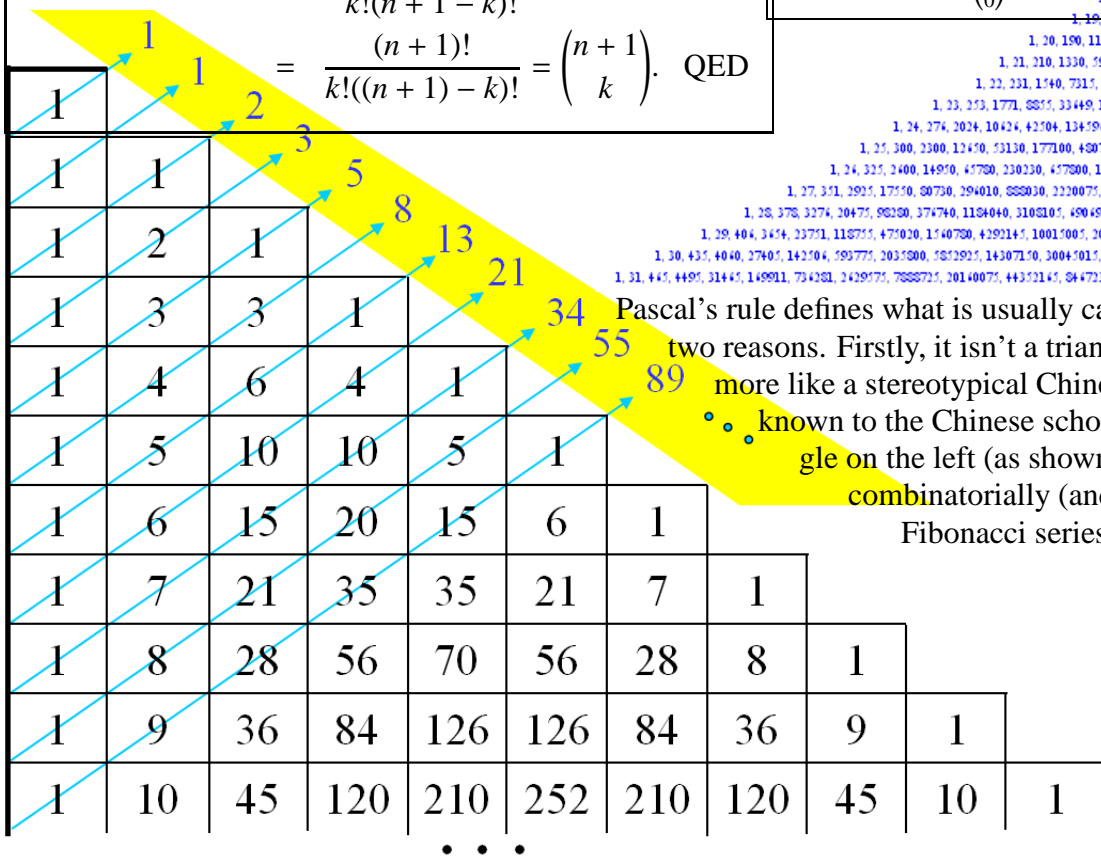
Pascal's rule defines what is usually called Pascal's triangle, presented as shown above. However, this is a misnomer for two reasons. Firstly, it isn't a triangle at all, unless font size decreases quadratically with increasing row number; it is more like a stereotypical Chinese hat! Which is appropriate enough because, secondly, this triangle and rule were known to the Chinese scholar Jia Xian, six hundred years before Blaise Pascal. Aligning the rows of the triangle on the left (as shown on the left) seems to make much better sense, typographically, computationally & combinatorially (and is how Pascal himself presented his triangle). A well-known relationship with the Fibonacci series, for instance, becomes immediately apparent as a series of diagonal sums.

The work of Jia Xian has passed to us through the commentary of Yang Hui (1238-1298) and Pascal's triangle is known in China as 'Yang Hui's triangle'. In Iran, it is known as the 'Khayyám triangle' after Omar Khayyám (1048-1131), although it was known to Persian, and Indian, scholars in the tenth century. Peter Cameron cites Robin Wilson as dating Western study of Pascal's triangle as far back as the Majorcan theologian Ramon Llull (1232–1316).