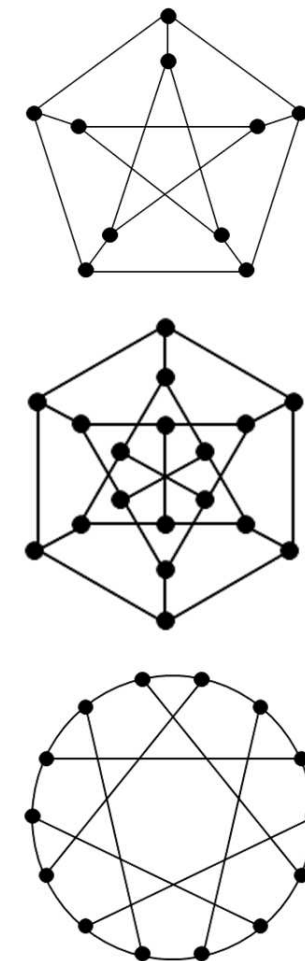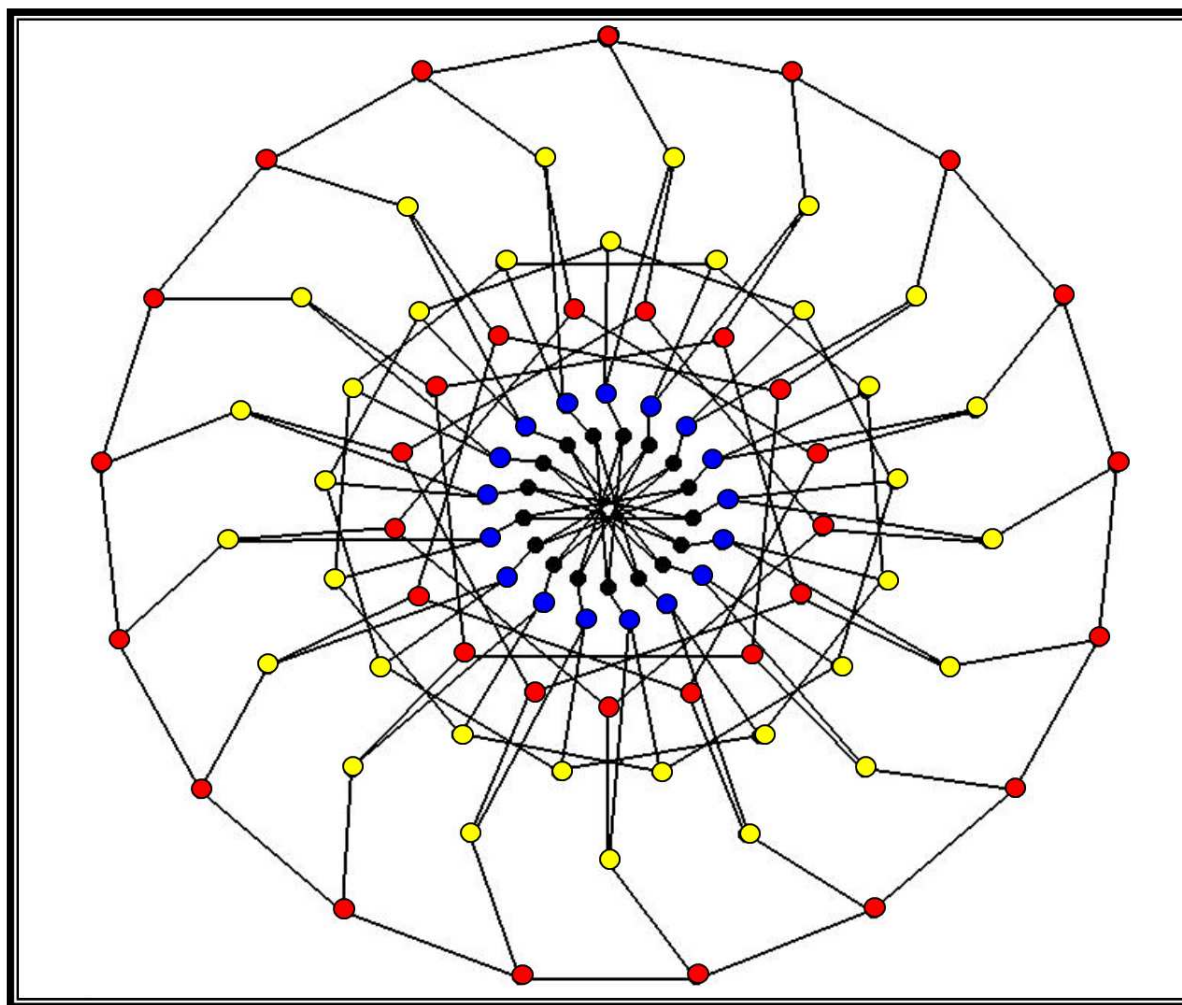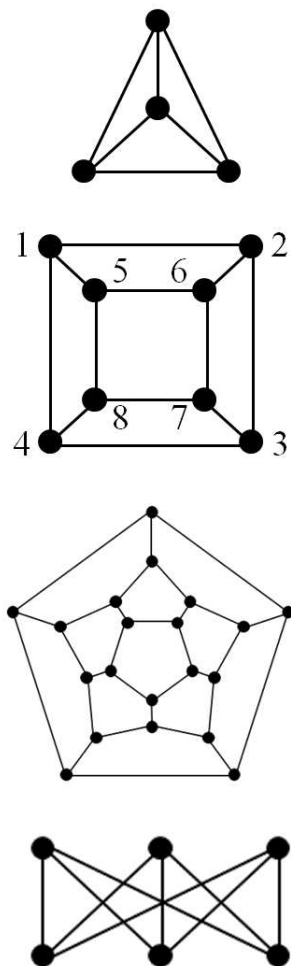# THEOREM OF THE DAY

Cameron's Theorem on Distance Transitive Graphs *For any integer $k > 2$ there exist only finitely many finite distance-transitive k-regular graphs.*

Distance transitivity means: if vertices a and b are the same (minimum) distance apart as α and β then there is a permutation of all the vertices which takes a to α and b to β and which preserves all edge relationships. For example, the graph of the cube is shown, labelled, above left. Vertices 1 and 7 are at distance 3 and so are vertices 4 and 6. And indeed the permutation (1 4)(2 3)(5 8)(6 7) preserves edge relationships while interchanging the positions of the vertex pairs $(1, 7)$ and $(4, 6)$. A number of distance-transitive 3-regular graphs (every vertex adjacent to precisely 3 others) are shown above. The Biggs–Smith graph in the centre is the largest. It has 102 vertices which have been colour-coded here according to their distance from the innermost circle of 17 (black) vertices.

Norman Biggs and Derek H. Smith proved in 1971 that there are exactly twelve 3-regular distance transitive graphs. It is at first sight very surprising that even a strong condition on symmetry should defeat the variety available in arbitrarily large graphs. Peter Cameron's deep theorem (1982) shows that this defeat applies even when arbitrarily many adjacencies are allowed.