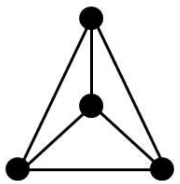
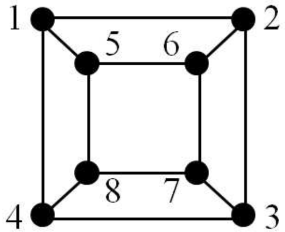
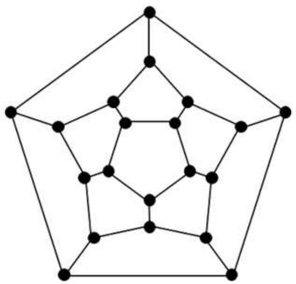
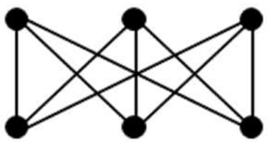
THEOREM OF THE DAY

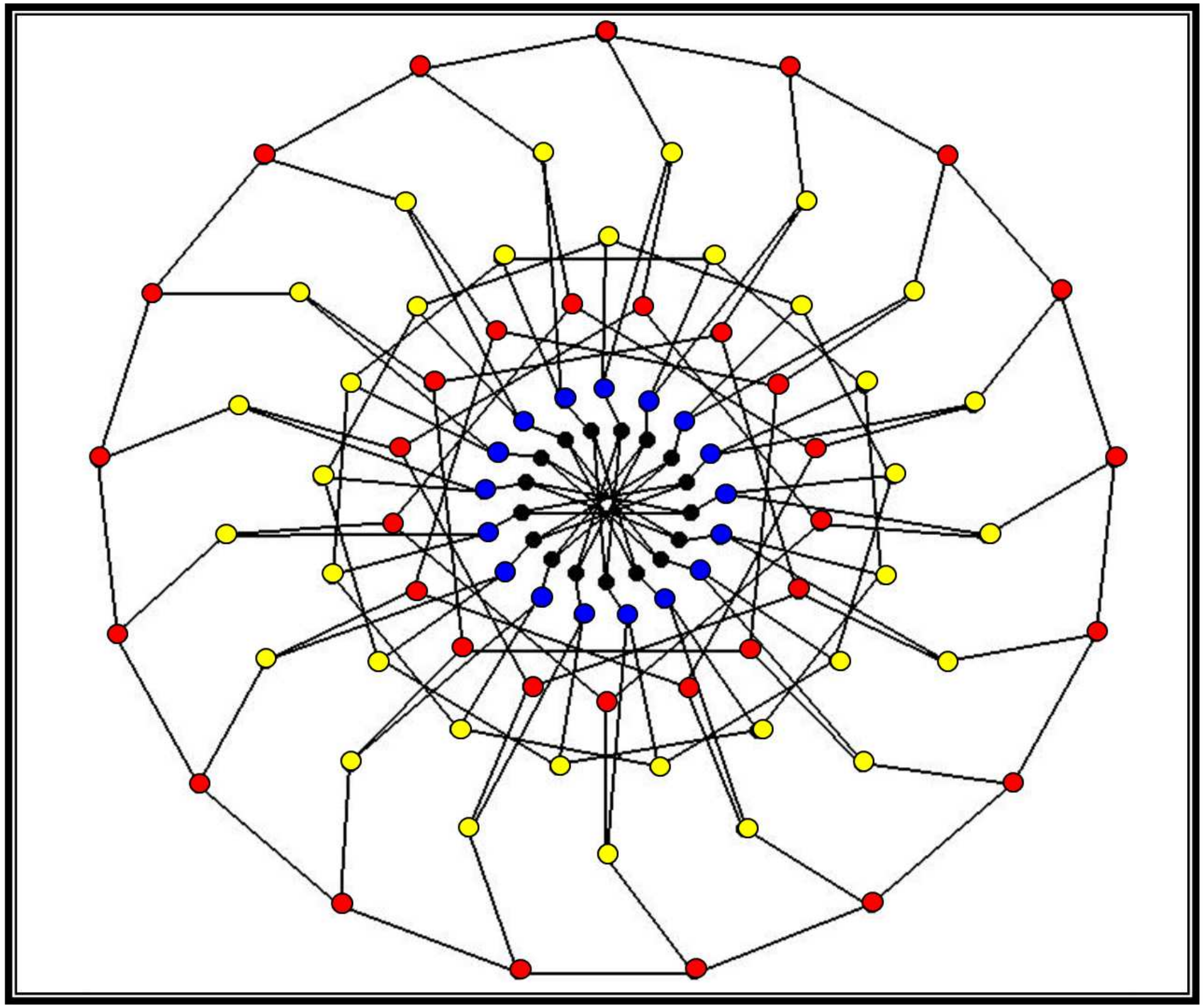
Cameron's Theorem on Distance Transitive Graphs *For any integer $k > 2$ there exist only finitely many finite distance-transitive k-regular graphs.*

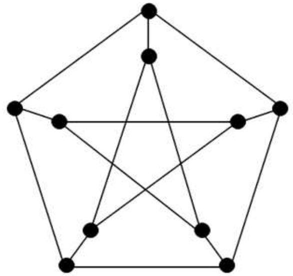


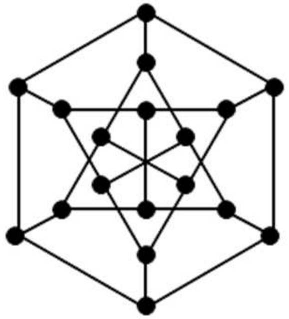


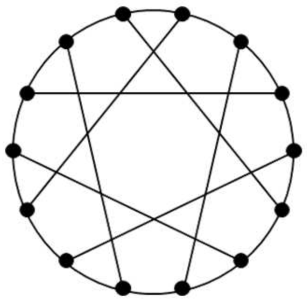












Distance transitivity means: if vertices a and b are the same (minimum) distance apart as α and β then there is a permutation of all the vertices which takes a to α and b to β and which preserves all edge relationships. For example, the graph of the cube is shown, labelled, above left. Vertices 1 and 7 are at distance 3 and so are vertices 4 and 6. And indeed the permutation (1 4)(2 3)(5 8)(6 7) preserves edge relationships while interchanging the positions of the vertex pairs (1, 7) and (4, 6). A number of distance-transitive 3-regular graphs (every vertex adjacent to precisely 3 others) are shown above. The Biggs–Smith graph in the centre is the largest. It has 102 vertices which have been colour-coded here according to their distance from the innermost circle of 17 (black) vertices.

Norman Biggs and Derek H. Smith proved in 1971 that there are exactly twelve 3-regular distance transitive graphs. It is at first sight very surprising that even a strong condition on symmetry should defeat the variety available in arbitrarily large graphs. Peter Cameron's deep theorem (1982) shows that this defeat applies even when arbitrarily many adjacencies are allowed.

Web link: www.math.mun.ca/distanceregular/

Further reading: *Algebraic Graph Theory (2nd Edition)* by Norman Biggs, C.U.P., 1994.