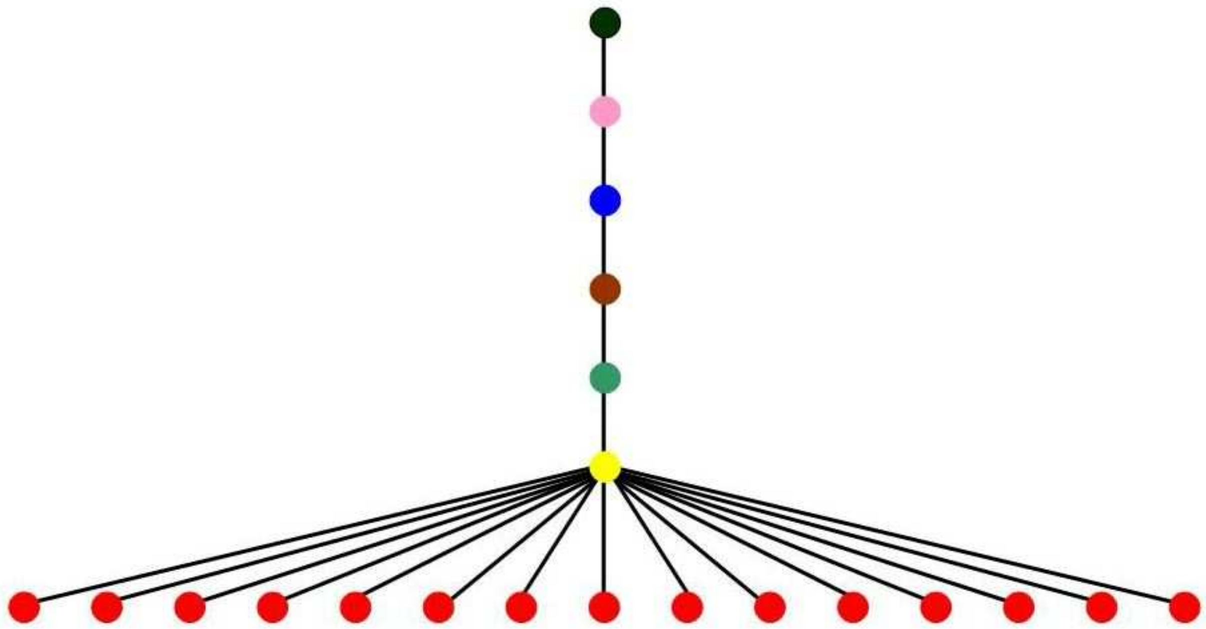
THEOREM OF THE DAY

Dilworth's Theorem *The size of a maximum antichain in a partial order is equal to the minimum number of chains needed to cover its elements.*



The 21 balls of snooker constitute a partially ordered set (or 'poset'): the 'colours' (non-red balls) are *totally ordered* since every colour is worth either more or less than any other. The 15 red balls, on the other hand, can be compared to the colours but are incomparable with respect to each other. They constitute the maximum antichain: the largest set of mutually incomparable elements. An equally-sized set of 15 chains is given by the vertical paths from each red ball up through the colours. These paths have length 7, which is the *length* of the partial order. Dilworth's theorem may equivalently be stated as: if a partial order has $ab + 1$ elements then there is a chain of size $a + 1$ or an antichain of size $b + 1$. For the $21 = 4 \times 5 + 1$ snooker balls, both are easily obtained.

CALTECH's Robert Dilworth was one of the pioneers of lattice theory. His 1950 theorem generalises a classic 1935 result of Erdös and Szekeres: any sequence of $ab + 1$ real numbers contains either a non-decreasing subsequence of length $a + 1$ or a non-increasing subsequence of length $b + 1$. E.g., 1 3 2 4 3 has $2 \times 2 + 1$ numbers; the longest non-increasing subsequences have length 2: 3 2, 3 3, and 4 3; but there are non-decreasing subsequences of length 3: 1 3 4, 1 3 3, 1 2 4, 1 2 3.

Web link: en.wikipedia.org/wiki/Dilworth's_theorem

Further reading: *Introduction to Lattices and Order* by B.A. Davey and H.A. Priestly, Cambridge University Press, 1990.