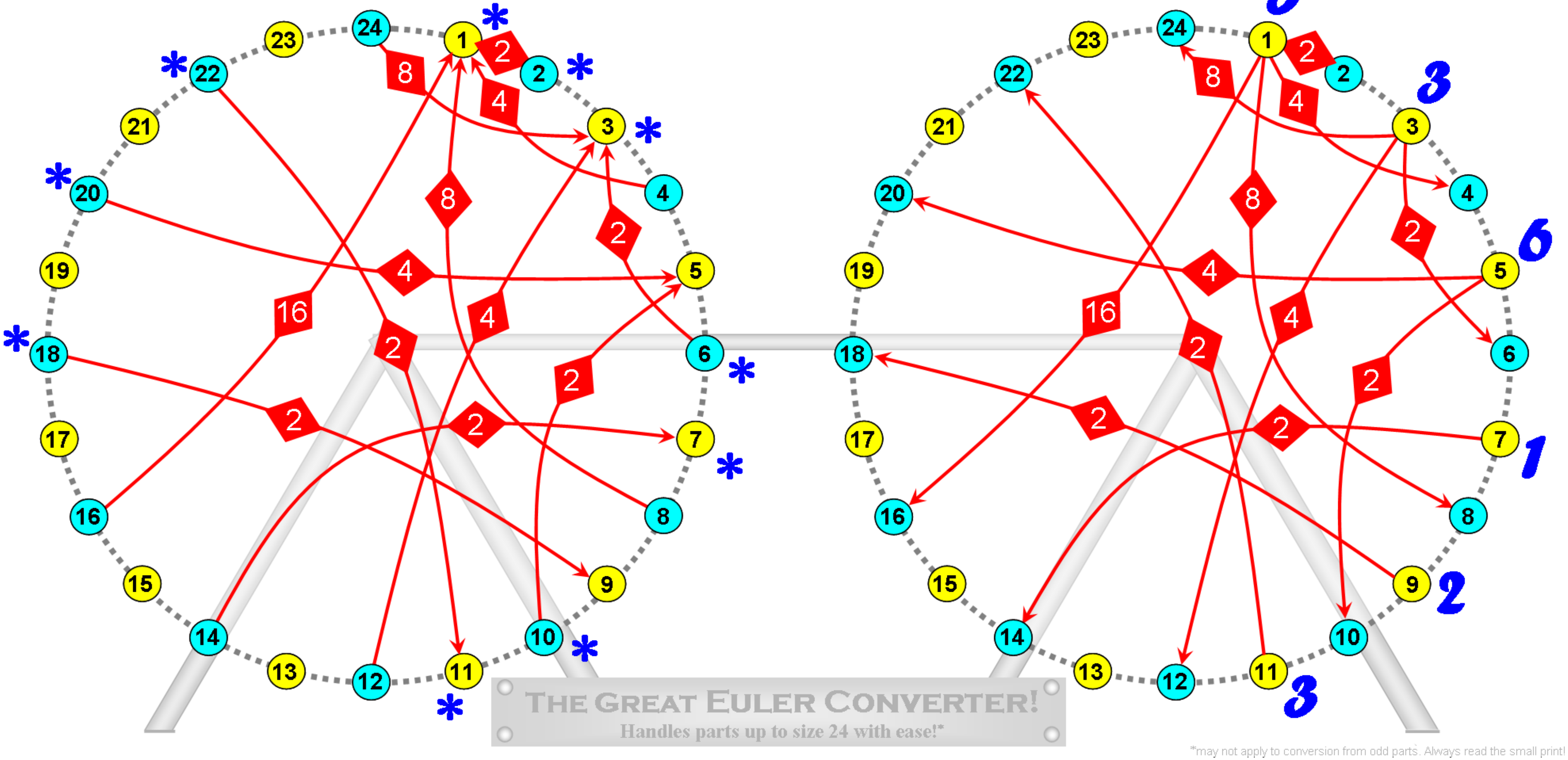
THEOREM OF THE DAY

Euler's Partition Identity *The number of partitions of a positive integer n into distinct parts is equal to the number of partitions of n into odd parts.*



The Converter. Left: distinct parts → odd parts. Example input: partition of $n = 100$ into distinct parts: $1+2+3+6+7+10+11+18+20+22 = 100$. Replace each even entry with copies of the (odd) number to which it has an arrow, the number of copies being indicated by the arrow, e.g. 20 is replaced by four copies of 5. Odd entries are unchanged. Output: $1+1+1+3+3+3+7+5+5+11+9+9+5+5+5+5+11+11 = 100$.

Right: odd parts → distinct parts. For each odd number write its frequency as a sum of powers of two. Include in the new partition the numbers indicated by the arrows, including the odd number itself if its frequency is odd. E.g. frequency of 5 is $6 = 2+4$ and the corresponding arrows point to 10 and 20.

The Converter illustrates a bijection confirming Leonhard Euler's 1748 identity by putting its two sides into correspondence.

Web link: shreevatsa.wordpress.com/2008/10/15/

Further reading: *Integer Partitions, 2nd revised ed.* by George E. Andrews and Kimmo Eriksson, Cambridge University Press, 2004, chapter 2.