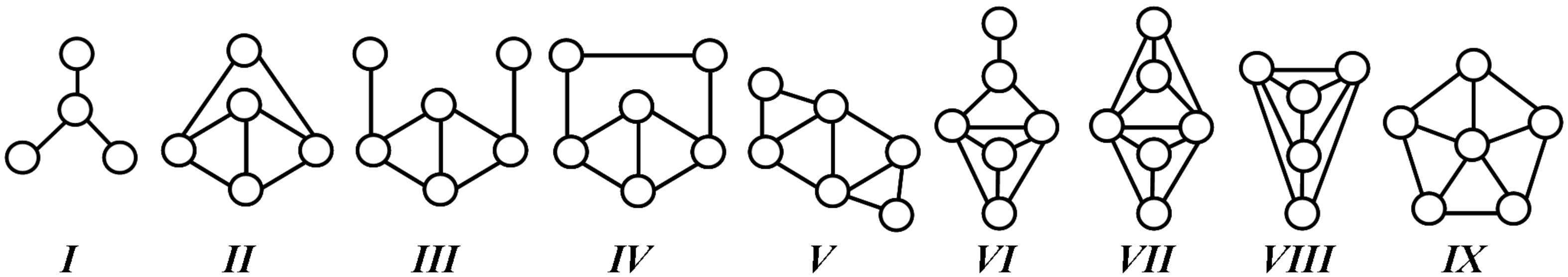
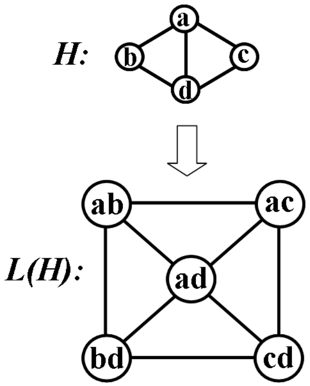
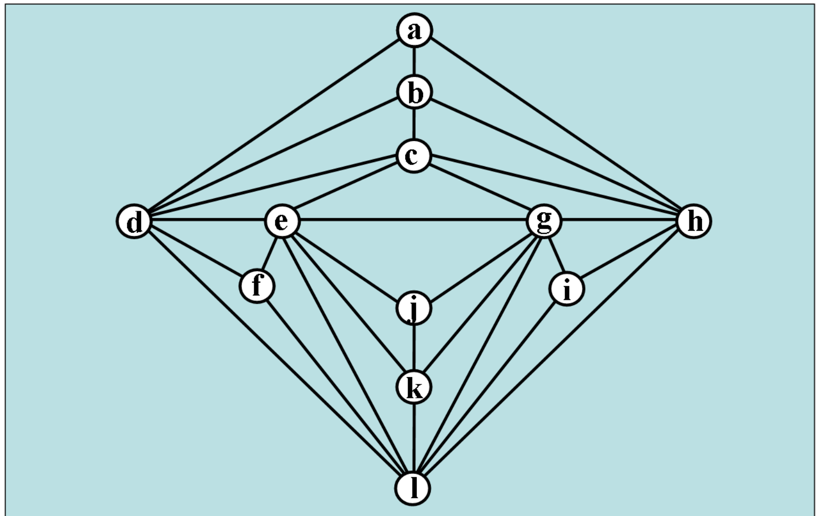
# THEOREM OF THE DAY

**Beineke's Theorem on Line Graphs** *Let G be a graph. There exists a graph H such that G is the line graph of H if and only if G contains no induced subgraph from the following set:*




For a graph $H$, the line graph $L(H)$ has a vertex for every edge of $H$ and an edge for every pair of incident edges of $H$.




**A very 'nonlinear' graph**

**A dedication for today's theorem:**

To the mathematician Lowell Beineke,
Who characterised the graphs leineke
Thus: it is forbidden
In *G* to find hidden
A graph from his set of size neineke.

An *induced subgraph* is a subset of the vertices of *G* together with each and every edge of *G* joining any two vertices of this subset. The 'very nonlinear' graph above contains every one of Beineke's forbidden set (see how quickly you can find them; the tricky bit is that they must be *induced* so that *a*, *b*, *d*, *h*, for example, will not do for graph *I* whose three end-vertices cannot be adjacent to each other).

Line graphs are a fundamental construction in graph theory. For example, edge-colourings of a graph *G*, where incident edges must be differently coloured, are the same as vertex colourings of $L(G)$, where adjacent vertices are differently coloured. In this 1968 characterisation of line graphs, due to Lowell Beineke and, independently, Neil Robertson, the forbidden induced subgraph I is known as a 'claw'. The property of being 'claw-free', possessed by line graphs, is again fundamental in graph theory.

**Web link:** www.newton.ac.uk/event/csmw06/seminars: see Graphs and Matroids I, by Bill Jackson.

**Further reading:** *Graph Theory* by Frank Harary, Perseus Books, 1999.