The Marriage Theorem

In a set of $n$ women each specifies a list of the men she is prepared to marry, as a subset, $W_i$, of a set of $n$ men, $i = 1, \ldots, n$. Assuming that any man will accept any offer of marriage then there is a monogamous espousal matching woman $i$ with a man from $W_i$, $i = 1, \ldots, n$, if and only every subset $X$ of women like a combined total of at least $|X|$ men.

Hall’s Marriage Theorem extends, more generally, to a theorem about finding a transversal for a (possibly uncountable) collection of finite subsets of a set: a representative from each subset with no two representatives the same.

Two theorems for the price of one! And, in fact, these superficially different results, both minimizing an obstacle to achieve a maximum result, belong to a collection of essentially equivalent minimax theorems which were discovered independently, in this instance by Philip Hall in 1935, and by Georg Frobenius (1917) and Dénes König (1931), respectively.

Web link: home.cc.umanitoba.ca/~borgerse/Presentations/GS-05R-1.pdf