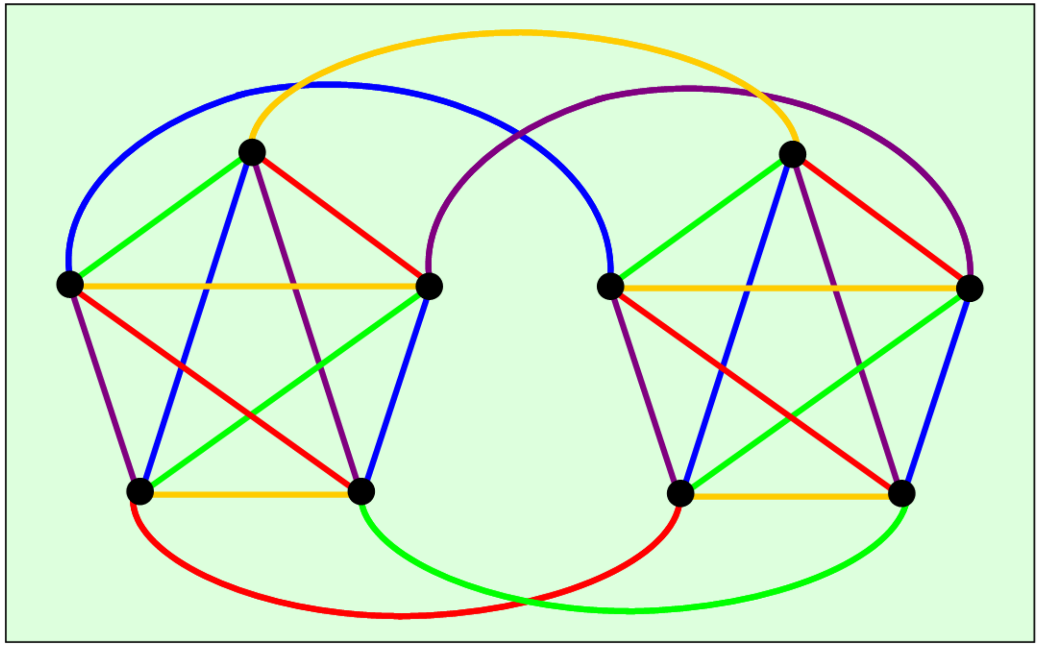
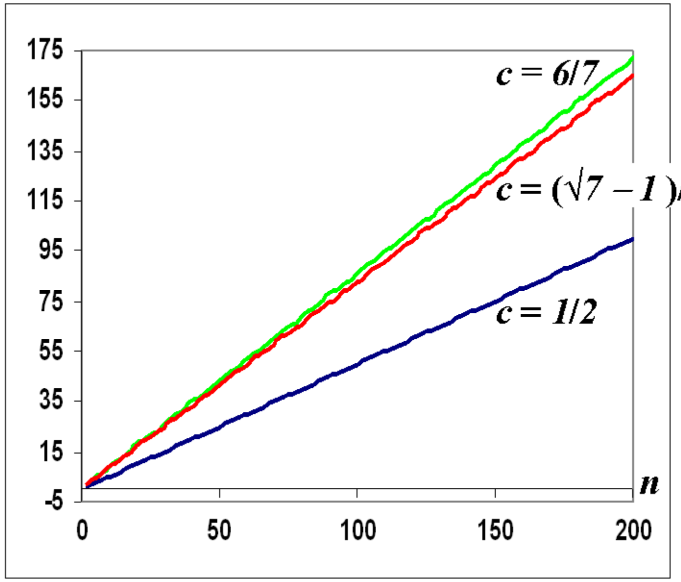
THEOREM OF THE DAY

1-Factorisation of Regular Graphs (a Theorem Under Construction!) *There exists a constant, c, such that all simple d-regular graphs of even order, n, with $cn \leq d$, have a 1-factorisation.*

Colour-free version



A 1-factorisation of $K_2 \times K_5$ ($n = 10$, $d = n/2$)



Comparison of cn with increasing n

The graph $K_2 \times K_t$ (that is, two copies of K_t with edges joining corresponding vertices) has a 1-factorisation for all t with $1 \leq t$, and is d-regular with $d = cn$, $c = 1/2$ (that is, every vertex is incident with $n/2$ edges). This is illustrated, above left, for $t = 5$ (with, what is more, a *perfect* 1-factorisation: any pair of edge-colours produces a Hamiltonian circuit in the graph); but the ultimate goal of $c = 1/2$ is well below what has been achieved, so far, for general d-regular graphs, as the right-hand chart shows.

Construction notes:

1985: Amanda Chetwynd and Anthony Hilton prove existence of c by showing $c \leq 6/7$. They conjecture that $c = 1/2$ is best possible.

1985: R. Häggkvist proves $\forall \varepsilon$, can take $c = 1/2+\varepsilon$ for large enough n ('97: published independently, Perković & Reed).

1989: Chetwynd and Hilton achieve $c = (\sqrt{7}-1)/2 \approx 24/29$ (as do Niessen and Volkmann independently, 1990).

2004: Hilton's student David Cariolaro achieves $c = (\sqrt{57}-3)/6 \approx 22/29$, except for 2 special classes of d-regular graphs.

2013: Bèla Csaba, Daniela Kühn, Allan Lo, Deryk Osthus and Andrew Treglown prove the conjecture for large n (removing the ε from Häggkvist and Perković & Reed's 1985 result).

Web link: Allan Lo's talk (1MB pdf) from www.maths.dur.ac.uk/events/Meetings/LMS/2013/GTI13/talks.html

Further reading: *Graph Edge Coloring: Vizing's Theorem and Goldberg's Conjecture* by Michael Stiebitz, Diego Scheide, Bjarne Toft and Lene M. Favrholdt, Wiley-Blackwell, 2012, Chapter 9.