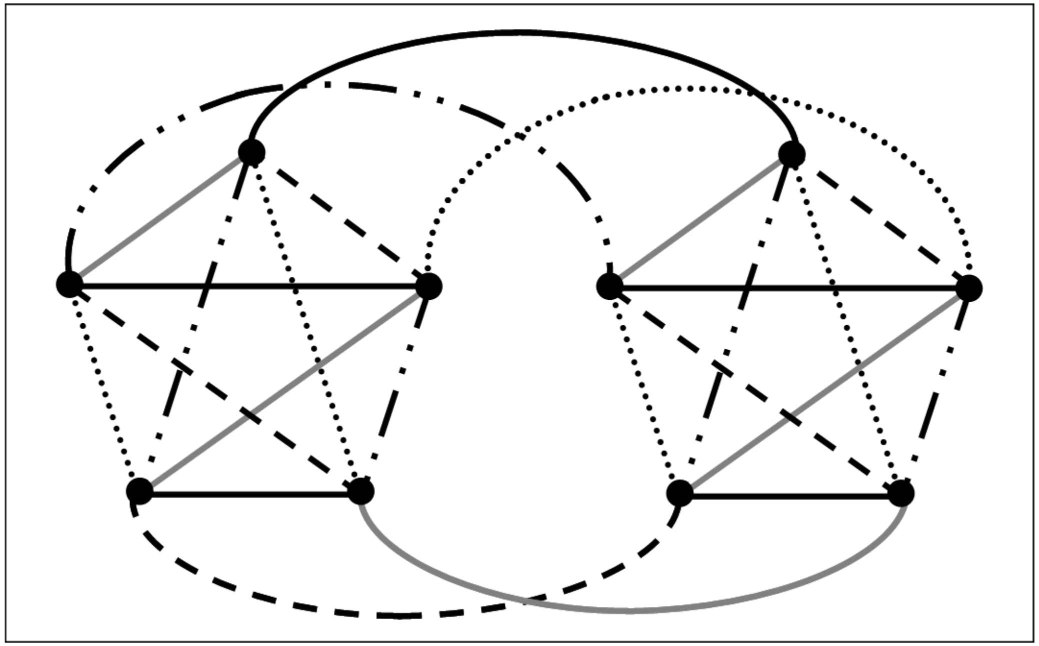
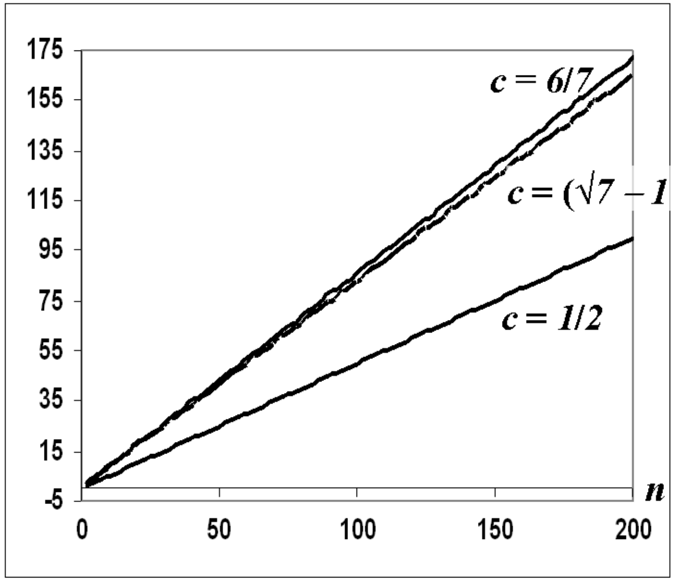
# THEOREM OF THE DAY

**1-Factorisation of Regular Graphs (a Theorem Under Construction!)** *There exists a constant, c, such that all simple d-regular graphs of even order, n, with $cn \le d$, have a 1-factorisation.*



**A 1-factorisation of $K_2 \times K_5$ ($n = 10$, $d = n/2$)**




**Comparison of $cn$ with increasing $n$**

The graph $K_2 \times K_t$ (that is, two copies of $K_t$ with edges joining corresponding vertices) has a 1-factorisation for all $t$ with $1 \le t$, and is $d$-regular with $d = cn$, $c = 1/2$ (that is, every vertex is incident with $n/2$ edges). This is illustrated, above left, for $t = 5$ (with, what is more, a *perfect* 1-factorisation: any pair of edge-colours produces a Hamiltonian circuit in the graph); but the ultimate goal of $c = 1/2$ is well below what has been achieved, so far, for general $d$-regular graphs, as the right-hand chart shows.

**Construction notes:**

1985: Amanda Chetwynd and Anthony Hilton prove existence of $c$ by showing $c \le 6/7$. They conjecture that $c = 1/2$ is best possible.

1985: R. Häggkvist proves $\forall \varepsilon$, can take $c = 1/2 + \varepsilon$ for large enough $n$ ('97: published independently, Perković & Reed).

1989: Chetwynd and Hilton achieve $c = (\sqrt{7}-1)/2 \approx 24/29$ (as do Niessen and Volkmann independently, 1990).

2004: Hilton's student David Cariolaro achieves $c = (\sqrt{57}-3)/6 \approx 22/29$, except for 2 special classes of $d$-regular graphs.

2013: Bèla Csaba, Daniela Kühn, Allan Lo, Deryk Osthus and Andrew Treglown prove the conjecture for large $n$ (removing the $\varepsilon$ from Häggkvist and Perković & Reed's 1985 result).

**Web link:** Allan Lo's talk here: www.maths.dur.ac.uk/events/Meetings/LMS/2013/GTI13/talks.html

**Further reading:** *Graph Edge Coloring: Vizing's Theorem and Goldberg's Conjecture* by Michael Stiebitz, Diego Scheide, Bjarne Toft and Lene M. Favrholdt, Wiley-Blackwell, 2012, Chapter 9.