# THEOREM OF THE DAY

A Theorem on Rectangular Tensegrities *Let L be a rectangular framework consisting of m rows and n columns of square cells constructed from rigid rods connected to each other by pivot joints, allowing rotation of the rods in the plane. Now suppose that in certain of the square cells 'non-stretching' cables are placed between either or both pairs of diagonally opposite points, preventing these pairs from moving further apart. Then the framework L is rigid in the plane if and only if the following bipartite directed graph D(L) is strongly connected: D(L) has a vertex for each row and column and row i has a directed edge to (from) column j if and only if the corresponding cell has a top-left to bottom-right (bottom-left to top-right) cable.*

Is this framework rigid? In more formal terms we are asking if any continuous transformation of the structure must be isometric (distance preserving). Given that cables can bend, the answer is NO and indeed, the transformation on the far right is clearly not isometric. How can we add extra cables to prevent this? The theorem gives an unexpectedly discrete answer to this continuous problem.
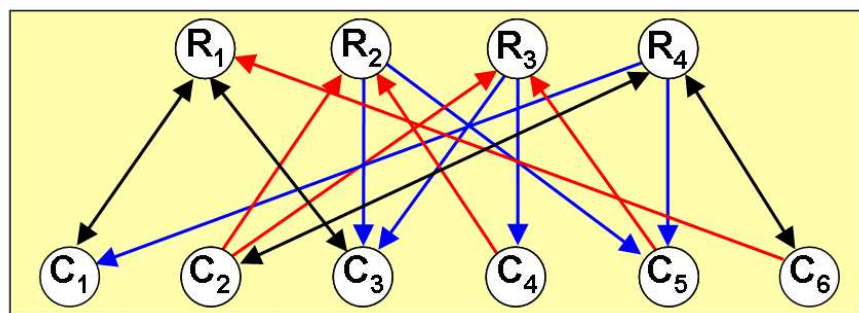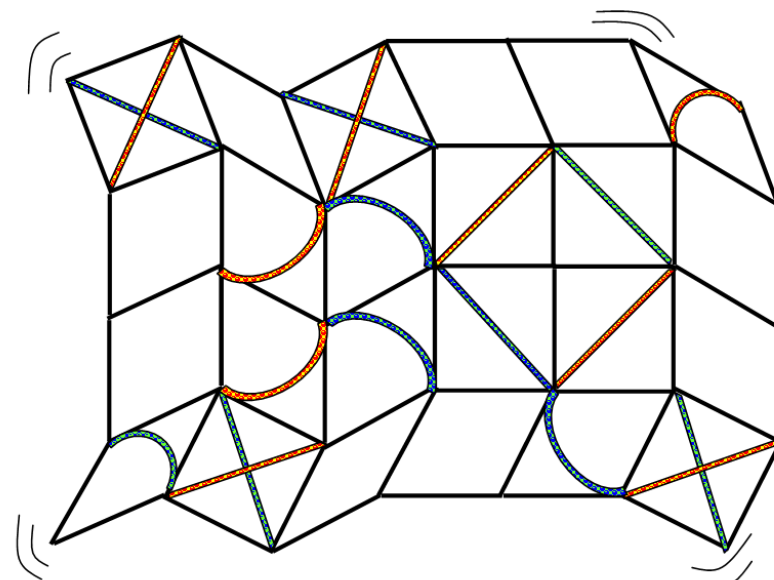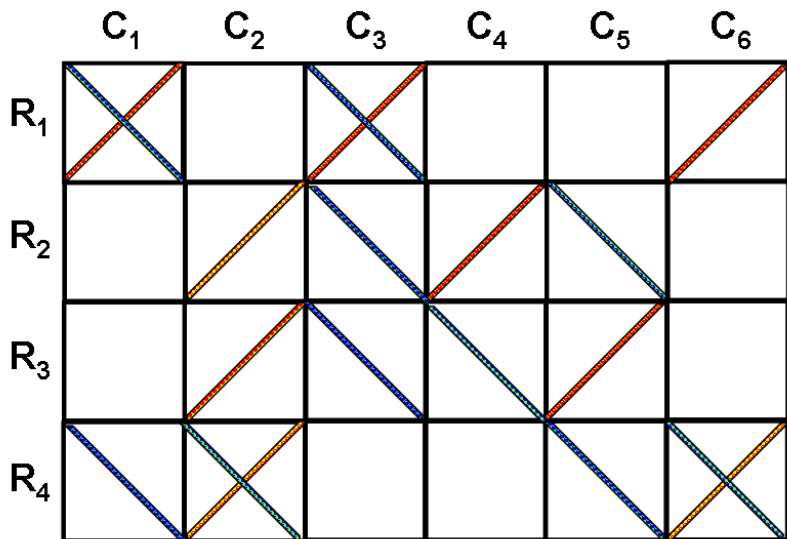
On the left we construct the digraph $D(L)$ specified by the theorem: bottom-left-top-right cables (red) are directed upwards; top-left-bottom-right cables (blue) are directed downwards; the black double-headed arrows represent cells in which both red and blue cables are present. $D(L)$ is *strongly connected* if any vertex can reach any other along a consecutive sequence of directed edges. And we can see that this condition fails because no edges exit from the vertex set $\{R_1, C_1, C_3\}$. However, a blue edge directed from R_1 to C_2 is enough to bestow strong connectivity; so a blue cable in the cell in row 1 and column 2 will make the framework rigid.

Tensegrity was the name given by Buckminster Fuller to frameworks of rods and cables. The undirected version of this theorem, in which diagonal cables are replaced by rigid rods, was discovered by Ethan Bolker and Henry Crapo in 1977. The generalisation stated above is due to Jenny Baglivo and Jack Graver in 1983.