Sof’ya Vasil’yevna Kovalevskaya

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Plate 1 Sof'ya Vasil'yevna Kovalevskaya (nee Korvin-Krukovskaya), 1887. Born in Moscow, 15 January, 1850. Died in Stockholm, 10 February, 1891.

Plate 2.
Sof'ya Vasil'yevna
Kovalevskaya, 1868, aged 18.

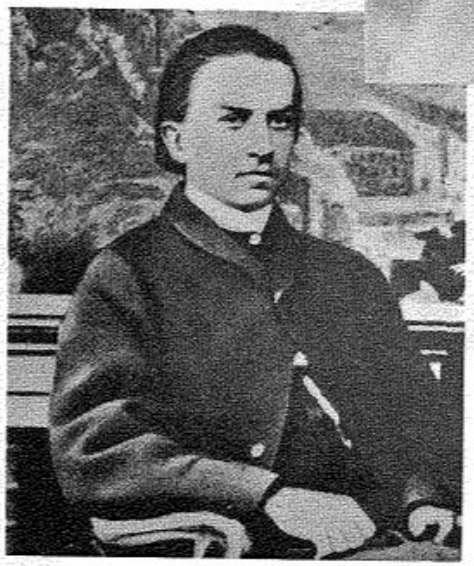

Plate 3.
Vladimir Onufriyevich
Kovalevskiĭ, (1842-1883),
aged 26.

Plate 4
Karl Theodore
Wilhelm Weierstrass,
(1815-1897), aged 70.

Plate 5
Soviet postage stamp
commemorating Kovalevskaya,
issued in 1951.

Plate 6 Sof'ya Vasil'yevna Kovalevskaya and Yulya Vsevolodovna Lermontova.

Plate 7 Kovalevskaya and her daughter, Sof'ya Vladimirovna (1878-1952), in 1885.

SOF'YA VASIL'YEVNA KOVALEVSKAYA*

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(received 28 June, 1976)

To say that Sof'ya Vasil'yevna Kovalevskaya (Plate 1) was the first woman mathematician is not entirely correct, since a few women had done some work in mathematics before her.

(1) Hypatia, daughter of Theon of Alexandria, wrote commentaries on the works of Ptolemy, Apollonius and Diophantos. She was lynched by Christians in the Cathedral of Alexandria in 415, and her works have not survived. [5].

(2) Voltaire's mistress Emilia, Marquise du Chatelet (1706-1749) translated Newton's *Philosophiae Naturalis Principia Mathematica* into French, and wrote commentaries on it. [5]

(3) Maria Gaetana Agnesi (1718-1799) wrote the *Institutions of the Differential Calculus* (in Italian), which became the standard textbook on calculus for the second half of the 18th century. By special Papal dispensation she was appointed Professor of Mathematics and Natural Philosophy at Bologna University, but she never lectured there. [5]

(4) Sophie Germain (1776-1835) earned the esteem of Gauss for her work in number theory, and was awarded a prize by the Paris Academy of Sciences for her work in the theory of vibrating plates. [5]

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(5) Byron's daughter Ada Augusta, Countess of Lovelace (1815-1851) collaborated with Charles Babbage in the planning of his proposed Calculating Engines, and published the first treatise on computer programming in 1842. ([27] p.xxi)

Nonetheless, Sof'ya Vasil'yevna Kovalevskaya was the first mathematician to be remembered primarily for achievement in mathematics, who happened to be a woman. She was born in Moscow, on 15 January 1850, into the Korvin-Krukovskiĭ family. Her mother Yelizaveta Fedorovna (née Shubert) (1820-1879) came from a line of German astronomers, and her father General Vasiliĭ Vasil'yevich Korvin-Krukovskiĭ (1800-1875), then commandant of the Moscow artillery garrison, came from a line of Cossack generals. She had an elder sister Anna (1844-1887), and a younger brother Fëdor (1855-1919). ([33] Ch.1)

After holding a succession of military posts, General Korvin-Krukovskiĭ retired in 1858 to his family estate of Palibino in the Vitebsk district of Belorussia, near the Lithuanian and Polish borders. There ensued a prolonged comedy, in which the General repeatedly paid gold to the College of Heralds to prove that he was really descended from the ancient nobility. On his 9th application, in 1869, the College confirmed his nobility, and his previous name of Kryukovskoĭ became the more sonorous Korvin-Krukovskiĭ. After 16 more years, Imperial approval was finally gained for a grandiloquent coat of arms. An imaginative family tree, showing the family's descent from the Kings of Hungary, was painted in bright colours on the wall of the library. ([33] p.11)

The General was a traditional patriarch, highly cultured but stern and autocratic. Sof'ya's Mama, twenty years younger than the General, was likewise highly cultured, but she was shy and retiring, easily dominated by her household serfs. In Sof'ya's infancy the

major influence on her was her Nurse, who convinced Sof'ya that her parents did not love her, and this conviction darkened much of Sof'ya's life.

The children were taught by a Polish tutor, and Anna had had the customary French governess, but from the age of 8 to 13 Sof'ya was in the charge of an English governess. Miss Margaret Smith exerted a powerful and lasting influence on her "wild young Cossack", whom she strove to convert into an English lady. After 5 years she admitted defeat, but she had gained the respect of the entire family by her efforts to impose some English order on a chaotic Russian feudal household.

Sof'ya's interest in mathematics was first aroused by the conversation of an uncle, who had no technical knowledge of mathematics but who liked to philosophise about it, and who intrigued his young niece with his discussions of topics such as asymptotes to curves, and the squaring of the circle. ([17] Chapters 1-5.)

In her enchanting *Recollections of Childhood*, Chapter 5, [17], Sof'ya tells us that: "When we removed to our country estate the house had to undergo major repairs, and all of the rooms were re-papered. But we had many rooms, and there was not sufficient wall-paper for one of the children's rooms. Since we got all of our paper from St. Petersburg, it was not worthwhile to order a small amount, and hence the slighted room remained for a number of years pasted with some spare sheets of paper. By a happy chance those sheets turned out to be Ostrogradskiĭ's lithographed lecture notes on the differential and integral calculus, which my father had bought in his student days. Those sheets covered with strange, incomprehensible formulas, soon attracted my attention. I remember how, in my childhood, I used to spend hours on end before that mysterious wall, trying at least to decipher individual phrases and to discover the proper

order of the sheets. By dint of daily scrutiny, the formulas and even the wording became imprinted on my brain, although at the time they were Greek to me."

That was when she was 11 years of age. Five years later, when she began to be taught the calculus, her tutor was amazed to find that she grasped immediately the concepts of limit and derivative "just as if she had known them before". The memorized lecture notes now began to make sense!

In 1864 a neighbour, Professor Tyrtov, presented his text on *Elements of Physics* to the General. Sof'ya (then aged 14) began to study the text, and in the chapter on optics she came across the concept of "sine". She invented the concept of "chord", and used it to understand the text. Tyrtov called her "a new Pascal", and urged that she be taught higher mathematics. Her family eventually agreed that she should study with the renowned tutor Strannolyubskiĭ in St. Petersburg, when she visited the capital with her mother and sister. She also received some tuition from Chebyshev in St. Petersburg. ([19] p.125)

In the 1860's, Russian society was in a state of intense ferment. [6] In 1861 Tsar Aleksandr II had emancipated the serfs - a partial reform which satisfied no one, least of all the former serfs. Very many of the educated young people adopted the creed of Nihilism, which involved rejection of the entire system of society, politics, religion and authority, particularly parental authority. The Nihilists were distinguished by their enthusiasm for science, for education, for the advancement of women, and for the alleviation of the sufferings of the peasants. In the opinion of their elders, they were also noted for their addiction to the drinking of tea. Subsequently, Russians would speak of "a woman of the 60's", much as we speak of "a man of the 30's".

The social turmoil led to increasing political repression, with the suppression of all publications which were not totally subservient to the State, and with mass arrests and transportations, especially of students. In 1877, Sof'ya attended a mass trial of 193 students arrested for demonstrating. In reaction to oppression the Nihilists became more forcible in their opposition, and the situation became so strained that when Oscar Wilde published his first play *Vera* in 1880, his heroine led a band of Nihilists who assassinated the Tsar. A London production of *Vera* was in rehearsal in 1881, but the production was cancelled when Tsar Aleksandr II was actually assassinated by a band of Nihilists, who were led by Vera Figner! (A case of "Nature imitating Art" ?). [24]

Radical ideas were introduced to the isolated region of Palibino by the local priest's son, whose support for Darwin resulted in the priest spraying him with holy water and ordering him to leave the house. His lengthy discussions with Sof'ya's sister Anna eventually had the effect of converting her to Nihilism, and this led to her first open quarrel with her father. Sof'ya always adored her elder sister, and she shared Anna's newly-found radical ideas. ([17] Ch.8)

Being bored by her "incarceration" at Palibino, Anna secretly began writing stories and sold two of them to Dostoyevskiĭ, for publication in his magazine. When her father discovered what Anna had done, accepting money from that "ex-convict and journalist", there was a frightful uproar, with much shouting and stamping and screaming. Eventually, however, after the General had been persuaded to listen to Anna reading her stories, he relented to the extent of permitting Anna to meet Dostoyevskiĭ in St. Petersburg, chaperoned by her mother. ([17] Ch.9)

In St. Petersburg, Dostoyevskiĭ soon became a close friend of Anna, of Sof'ya and of their Mama. Sof'ya (then aged 15) worshipped

Dostoyevskiĭ (then aged 43), and Anna (then aged 21) greatly admired him. Little Sof'ya convinced herself that Dostoyevskiĭ loved her, but was utterly mortified to encounter him in the very act of proposing marriage to Anna! Sof'ya became furiously jealous, but to her amazement Anna declined the proposal. The girls remained life-long friends with Dostoyevskiĭ and with his secretary, whom he married 6 months later. ([17] Ch.10)

The University of St. Petersburg had permitted women to attend lectures in 1861, but the Government took fright at the enthusiastic response and closed the University - when it re-opened that "privilege" had been withdrawn. Some private courses of advanced study for women were organized, but the Government refused any public facility. In view of this situation, Sof'ya and Anna decided that they must go abroad to study. Several Russian women had already gone to Germany and Switzerland to study, principally in medicine.

The peculiarities of Russian law at that period resulted in the curious institution of the "nominal marriage", whereby a girl of advanced views would order a young man to marry her, shake hands and thank him politely after the ceremony, and then take the first train to Switzerland - as a married woman, her parents could not block her application for a passport. Anna's friend Zhanna Yevreĭnova also wished to study abroad, but her father (a general at the Imperial Palace of Peterhof) had told her that he would sooner see her in her grave than at a University. The three girls decided to find one husband - the other two should be able to accompany the married one abroad. First they visited a young professor whom they knew slightly, and asked him to marry one of them. He politely declined, and 15 years later he was twitted by Sof'ya for having turned down that triple proposal. ([22] p.7)

Sof'ya (Plate 2) then looked considerably younger than her 18

years. She was a woman of quite exceptional physical beauty, and many of her friends remarked on the near-hypnotic power exerted by her large and expressive eyes. The photograph in Plate 2 seems to have been posed to conceal the marked asymmetry of her face, with the left eye more prominent than the right. One of her friends complained that many of the photographs of Sof'ya had been re-touched, to make her face more symmetrical. She became short-sighted, and was sufficiently vain to refuse to wear spectacles, so that she often squinted markedly. She paid little attention to furnishings and clothing, and was often somewhat untidy in appearance.

In 1868, Anna and Zhanna met Vladimir Onufriyevich Kovalevskiĭ (Plate 3), then aged 26. At that time he was studying law, to become a genealogist in the College of Heralds, but he was busily engaged in translating and publishing scientific works by Darwin, Huxley, Agassiz, von Baer and other scientists. Indeed, he published translations of two of Darwin's books simultaneously with their English publication! Later, he became a distinguished scientist, a founder of the science of evolutionary palaeontology. His elder brother Aleksandr was a renowned zoologist, founder of the science of comparative embryology. Both brothers strongly supported Darwin's work on evolution, and they were admired by Darwin and Huxley. [5]

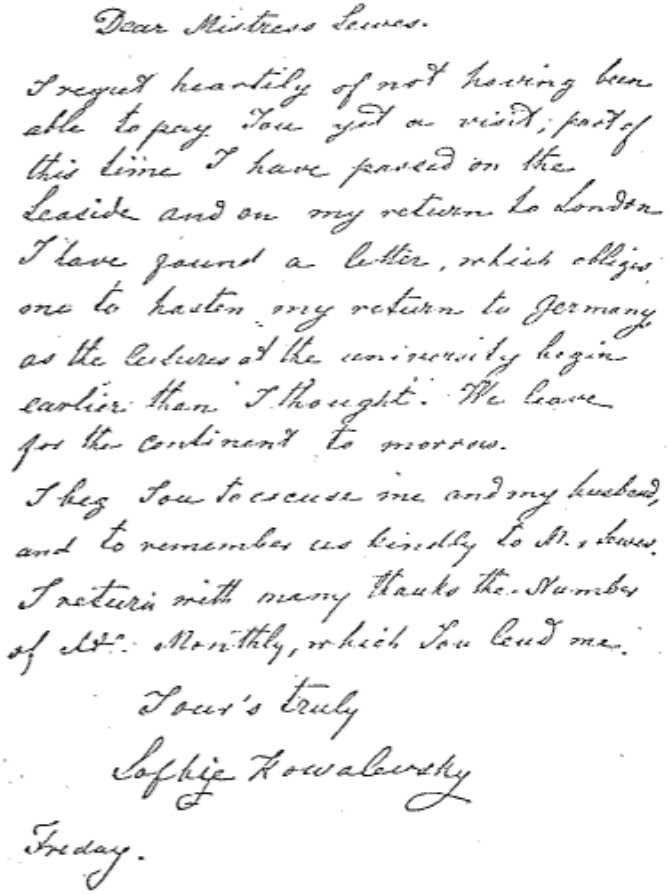
Vladimir agreed to marry either Anna or Zhanna, leaving the choice up to them. However, when he met Sof'ya he changed his mind, and insisted on marrying her. The "little sparrow" enchanted him - he wrote to her in May 1868 that: "you should look on me now, not as a man doing you a favour, but as a comrade striving jointly with you towards a single goal, i.e. I am just as necessary to you as you are to me - therefore, make use of me accordingly, and entrust to me whatever you may take into your head without fear of burdening me: I shall work for you, just as much as for myself". ([20] p.484)

After elaborate intrigues, her father's consent to the marriage was obtained. Immediately after the wedding on 15th September 1868 (Julian calendar), Sof'ya and Vladimir went to St. Petersburg to attempt to study there, and also to seek a similar husband for Anna. There were idyllically happy relations between Sof'ya and her devoted "brother". She wrote to Anna (17th September 1868) that she and Vladimir kept lamenting the fact that he wasn't a Mohammedan, and hence he couldn't marry both of them. ([20] p.223)

She managed to get special permission from two professors to attend their lectures on physiology and chemistry. By the end of 1868 she saw that it was impossible to spread her work so widely, and that she would need to concentrate on chemistry and mathematics. Meanwhile, Vladimir had decided to abandon the loathesome study of heraldry, and to concentrate on geology and palaeontology. Accordingly, in April 1869, Sof'ya and Vladimir went to Germany to study. Sof'ya eventually gained permission to study at Heidelberg, where for 3 semesters she studied chemistry with Bunsen, physics with Helmholtz and Kirchoff, mathematics with du Bois Reymond and Königsberger. The professors became enthusiastic about this shy, modest, brilliant Russian girl, whose fame soon spread : indeed, one day in the street a poor woman told her child "Look, look! There's the girl who's so good at school!" ([22] p.18). Sof'ya and Vladimir were joined by their friend Yulya Lermontova, who studied chemistry and shared their flat.

In 1869, Sof'ya and Vladimir visited England, to meet Darwin and Huxley. The novelist George Eliot (Mary Anne Evans, 1819-1880) had already heard of Sof'ya, and invited her to visit. At their first meeting, Sof'ya had a heated dispute with another guest about the intellectual capabilities of women, and was mortified to find that she had been disagreeing with Herbert Spencer. Sof'ya remained a close friend of George Eliot for the rest of her life, last visiting her

only 12 days before her death. ([21] pp.230-243)



Dear Mistress Lewes.

I regret heartily of not having been able to pay You yet a visit; part of this time I have passed on the Seaside and on my return to London I have found a letter, which obliges one to hasten my return to Germany as the lectures at the university begin earlier than I thought. We leave for the continent to morrow.

I beg You to excuse me and my husband, and to remember us kindly to Mr Lewes.

I return with many thanks the Number of Mr. Monthly, which You lend me.

Your's truly

Sophie Kowalevsky

Friday.

Figure 1

Letter from Kovalevskaya to George Eliot. Reproduced by permission of The Beinecke Rare Book and Manuscript Library, Yale University.

George Eliot lived openly with the writer George Henry Lewes, and hence Sof'ya addressed her as "Mistress Lewes" in a letter (Fig.1), written probably on 8th October 1869, which is the only letter between them known to survive.

After Sof'ya and Vladimir had returned to Yulya in Heidelberg they were joined by Anna and Zhanna. Indeed, Zhanna had left Russia without permission, being fired at by border guards as she escaped across a swamp. Vladimir vacated the flat to make way for the newcomers, but Anna objected to the favouritism which he showed to Sof'ya, by taking her for walks more frequently than he took any of the other 3 girls. Accordingly, Vladimir left for Jena, where his doctoral dissertation on the evolution of ungulates created a scientific sensation.

Du Bois Reymond's enthusiastic accounts of Weierstrass's teaching determined Sof'ya to study in Berlin with Weierstrass (Plate 4), then aged 55 years. Sof'ya first visited Weierstrass on 3rd August 1870. At that time he had no idea of her extreme youth since she was dressed formally, with a heavy veil. He did not take her seriously, but gave her some exercises to test her ability. A week later she showed him her solutions, and he was so impressed that he agreed to teach her, after he had checked with Königsberger that "the lady's character offers the requisite guarantees". ([26] p.135). It was quite impossible to enrol her at Berlin University, and so Weierstrass taught her as a private pupil, from 1870 to 1874.

She quickly became his favourite pupil. He repeated his lectures to her and discussed his research work with her. He declared that "regarding Kovalevskaya's mathematical studies, I can testify that I have had very few pupils who can be compared with her for industry, diligence and devotion to science." ([7] p.346). All of her 9 published mathematical papers [7] are written on themes by

Weierstrass, and he acknowledged that her remarks had inspired some of his own work. They remained close friends, and when Sof'ya died Weierstrass (then aged 76) was heartbroken, and burnt her letters to him. He did, however, give permission to Mittag-Leffler to make use of his letters to Sof'ya, after Weierstrass's death. ([26] p.136). Excerpts from his letters to Sof'ya have been published in several instalments since 1900, and the complete letters were finally published in 1973 [34].

Her studies with Weierstrass were interrupted by a dramatic interlude in Paris. Bismarck's policy had succeeded in provoking France to war against Prussia, on 19th July 1870. After the ignominious flight of the Emperor Louis Napoleon in September 1870, the Parisians organized their own defence against the besieging Prussian army, while Adolphe Thiers organized a Republican Government at Versailles. After Thiers had surrendered France, the Prussian army briefly occupied Paris. Thiers attempted to take over Paris, but he was resisted by the National Guard, which had defended Paris against the Prussian siege. Thiers then proceeded to conquer Paris, with much more vigour than he had shown in resisting the Prussians. On 22nd March 1871 the Paris National Guard organized the Commune, which defended Paris against the attacking French Republican army for 72 days. After the French Republic captured Paris, "men, women and children were ruthlessly massacred - 25,000 of them at Montmartre alone". ([1] p.110)

Meanwhile, Anna had become dissatisfied with student life in Heidelberg and had gone to Paris, where she plunged into revolutionary and literary activity. She continued to accept an allowance from her parents, and forwarded her letters to her parents through Sof'ya, so that the post-mark would not contradict her story that she was studying together with Sof'ya. On the 27th March 1871 she married Charles Victor Jaclard, a teacher of mathematics who had become one

of the leaders of the Commune [23], and she became an active and prominent member of the Commune.

Sof'ya became very concerned about Anna's safety, and so she and Vladimir made the perilous journey to Paris at the beginning of April 1871. They managed to infiltrate through the lines of the Prussian army still surrounding Paris, and then walked along the banks of the Seine until they found a boat, which they took to row across the river, whilst a dilatory sentry shouted challenges at them. After crossing the Seine, they entered Paris on 5th April 1871. They remained there for 38 of the 72 days of the Paris Commune, leaving on 12th May. Sof'ya and Anna nursed the wounded in an abandoned convent, with shells constantly bursting around them, whilst Vladimir took advantage of the circumstances to study the collections of vertebrate fossils in the museums of Paris. ([20] pp.520-521). Sof'ya later declared her intention of writing a book about the experiences of Anna and herself in the Paris Commune, but that never did get written. ([22] p.30)

Sof'ya returned to her studies in Berlin, but after the fall of Paris to the French Republic she became so concerned about Anna's safety that she and Vladimir returned to Paris. On the train to Paris, she was horrified to read a newspaper report that Anna had been arrested - a similar report in The Illustrated London News (17th June, 1871) describes Anna as a "rather elegant woman .. of Russian birth, and the wife of the Chief of the 17th Legion". Upon reaching Paris they found that Anna had still managed to escape arrest, but that Jaclard had been captured. Vladimir wrote to his brother that "very many of our good friends have been shot, or otherwise killed" ([20] p.521). Sof'ya and Vladimir promptly organized Anna's escape to Heidelberg and then endeavoured to help Jaclard, who had escaped being shot summarily, but expected to be transported to New Caledonia. Sof'ya intended that she and Anna should accompany Jaclard to New Caledonia, as Sof'ya Marmeladova had accompanied Raskol'nikov to his

Siberian exile [4]. Vladimir, however, advised Sof'ya to complete her mathematical studies - he would escort Anna to New Caledonia, and Sof'ya could join the three of them after she had gained her degree. ([33] p.121)

None of those poor innocents realized that Siberian exile was, in some ways, more humane than the French penal colony in New Caledonia! When the families of the exiles were permitted to go to New Caledonia, they were transported in the manner of African slaves being shipped to America! After the death of Thiers, the surviving exiles were amnestied in 1879. ([1] ch.8)

However, such heroically idealistic sacrifices did not become necessary. Sof'ya and Anna wrote to their father, confessing that Anna had grievously deceived him. The General, that stern autocrat (then aged 71), and his shy, retiring wife (then aged 51) came to Paris on the first train, bringing with them enough gold to arrange for the escape of their unexpected son-in-law. The details were prudently concealed, but it is known that Vladimir gave his own passport to Jaclard for his journey from Paris to Zurich, on 7th October, 1871. Thereafter the girls both adored their father, and he became reconciled to their husbands and to their radical views. When he died suddenly in 1875, the girls were utterly heartbroken. ([33] p.122)

After this exciting interlude, Sof'ya resumed her studies with Weierstrass in Berlin. She lived with Yulya Lermontova (Plate 6) in dismal lodgings, both working with ferocious intensity, Sof'ya at her mathematics and Yulya at her chemistry.

For some years Sof'ya did not tell Weierstrass of her marriage, and when Vladimir visited Berlin she would introduce him vaguely as "a relation". ([22] p.24)

When recuperating from an illness in 1873 she stayed with Anna

and Jaclard in Zurich, where she was much attracted by the mathematics lectures of Schwartz. When Schwartz informed her that Weierstrass had been appointed as Rector of Berlin University she thought that Weierstrass would no longer have time to devote to her, and that she should continue her studies with Schwartz. But Weierstrass anticipated her thoughts, and he wrote a remarkable letter to her on 20th August 1873 ([34] pp.26-28), in which he assured her that he would always find time to talk with her. He described the beautiful lake scenery at the Baltic resort of Rügen where he was staying, and expressed his regret that she was not there with him. "How splendid a pair we would make here! You, with your vivid imagination and I, stimulated and illuminated by your enthusiasm, could dream and think here of the many problems which we could solve - of finite and infinite spaces, of the stability of the solar system and of all the other great problems for the mathematics and physics of the future. But I have recently become reconciled to the fact that not every beautiful dream can be accomplished". Sof'ya was much perturbed by this extraordinary letter, and she decided to continue her studies with Weierstrass, rather than transfer to Schwartz.

During her study with Weierstrass she wrote three papers:

On the Theory of Partial Differential Equations [8],

On the reduction of a certain class of Abelian integrals of the 3rd rank to elliptic integrals [9],

Supplementary remarks and observations on Laplace's research on the form of Saturn's ring [12].

The so-called "Cauchy-Kovalevski Theorem" on the existence and uniqueness of solutions of differential equations is presented in [8]. Actually, neither she nor Weierstrass knew of the earlier work by Cauchy and others, and her work is much more general than Cauchy's. ([29] p.5). Weierstrass considered each of these 3 papers to be fully adequate for a doctoral dissertation, and at his strong urging

Göttingen University awarded her a Ph.D. (without examination) in July 1874. ([34] p.298). Yulya was also awarded a Ph.D. by Göttingen, for her work in chemistry.

In September 1874, Sof'ya and Vladimir returned in triumph to Russia. For the next 6 years she did little in mathematics, apart from presenting her paper on Abelian integrals to a Russian scientific congress in 1880. It proved to be quite impossible for her to get appointed to any post in Russia higher than that of infant mistress. An academy for women opened in St. Petersburg in 1878, and she was a member of its governing committee for many years. She became a celebrity in high society, and in literary circles. She engaged in journalism and published theatrical reviews, scientific surveys, book reviews and a first novel *Der Privat-Dozent* (since lost). Weierstrass sent long letters to her, including some of his most highly prized research, but she replied only rarely and tardily. From 1878 to 1880 she did not answer any of his anguished letters. ([33] ch.5, [26] pp.149-170, [34] pp.48-84)

Vladimir had likewise been unable to get a post, despite his brilliant scientific reputation, and he engaged in financial speculations on a vast scale, ostensibly to finance the scientific and political projects planned by Sof'ya and him. In 1880 he was finally appointed professor of palaeontology at Moscow University, but soon afterwards his finances collapsed and he became the managing director of an oil-well firm.

Meanwhile the marriage had finally ceased to be nominal, and in October 1878 Sof'ya coyly announced in her letters that "a mathematician is born". In fact, her daughter Sof'ya Vladimirovna Kovalevskaya (1878-1952) (Plate 7), known as "Fufa", became a doctor rather than a mathematician, and she played a prominent role in the celebrations of the centenary of her mother's birth in 1950, translating her mother's

Swedish letters and writings into Russian ([30], pp.144-154)

Sof'ya assisted Vladimir in his financial affairs, studying geology so that she could collaborate with him. However, she came to the conclusion that he was inextricably entangled in his financial problems, and the directors of his oil-well company deliberately fostered discord between Sof'ya and Vladimir. As a consequence, in October 1880 she decided to resume the study of mathematics.

After a gap of two years she wrote to Weierstrass from Moscow, telling that she would like to meet him again. In fact, she left Moscow without waiting for his reply (written on 28th October 1880) to arrive - if she had received it she might have hesitated over her plan to re-visit Weierstrass. As it was, she arrived in Berlin on 31st October 1880 and met Weierstrass at 3 p.m. ([26] p.174). From 1881 to 1883 she studied intensely in Berlin and Paris, working on the propagation of light in doubly-refracting crystals. She continued to correspond with Vladimir, and they met occasionally. She lived in drab lodgings and looked after Fufa part of the time, but for much of the time Fufa was cared for by Yulya, at her estate near Moscow. Mittag-Leffler had met Sof'ya at St. Petersburg in 1876 and had been profoundly impressed by her. In 1880 he had offered to help her to get an appointment to the mathematics faculty at Helsingfors University where he was professor, but his efforts on her behalf merely weakened his own position there. On 27th April 1881, Sof'ya was elected a member of the Moscow Mathematical Society.

Meanwhile Vladimir's company collapsed, and the legal responsibility lay with him, so that on 27th April 1883 he killed himself (with chloroform). When Sof'ya heard of his suicide she collapsed into a high fever for 5 days - on the sixth day she took pencil and paper and resumed working at mathematics. Her financial situation was now severe - she devoted much effort to rehabilitating Vladimir post-

humously, showing that he was not responsible for the company's collapse. For the rest of her life she continued paying off her husband's debts to sundry "highway bandits". ([33] pp.181-189)

In 1881 Mittag-Leffler had become a foundation professor of the new University of Stockholm, which had been founded to provide a liberal alternative to Uppsala University, then under the domination of the Lutheran Church. After intrigues of positively Byzantine complexity he succeeded in getting Sof'ya appointed as professor of mathematics, with a probationary period as un-paid privat-dozent. In November 1883 her paper *On the propagation of light in crystalline media* was published in Acta Mathematica, and she arrived in Stockholm.

There was immense controversy in Sweden over the appointment of the first woman ever to teach mathematics at a university. One Swedish newspaper proudly proclaimed that: "Today we do not herald the arrival of some vulgar, insignificant prince of noble blood. No, the Princess of Science, Madame Kovalevskaya, has honoured our city with her arrival!" ([31] p.50). On the other hand August Strindberg, playwright and male chauvinist pig par excellence, wrote that: "A female professor of mathematics is a pernicious and unpleasant phenomenon - even, one might say, a monstrosity; and her invitation to a country where there are so many male mathematicians far superior in learning to her can be explained only by the gallantry of the Swedes towards the female sex". ([32] p.289)

Sof'ya spent 8 happy and productive years at Stockholm. She gave her first lecture on 11th February 1884, was appointed Professor of Mathematics (for 5 years) on July 1st 1884, became an editor of Acta Mathematica in 1884, was appointed also to the Chair of Mechanics in 1885 (thereby becoming a "professor squared"), and in 1889 was appointed as Professor of Mathematics for life. ([33] pp.211-241). She was very popular with her students, and with most of the other

professors. Usually she lectured twice a week, for 2 hours each time. The titles of the courses which she taught were [25]:

1. Theory of Partial Differential Equations (Autumn 1884).
2. Weierstrass's Theory of Algebraic Functions (Spring 1885).
3. Elementary Algebra (Spring 1885).
4. Weierstrass's Theory of Abelian Functions (Autumn 1885 to Spring 1887).
5. Theory of Potential Functions (Spring 1886).
6. Theory of Motion of a Rigid Body (Autumn 1886 and Spring 1888).
7. Poincaré's Theory of the Curves Defined by Differential Equations (Autumn 1887 and Spring 1888).
8. Weierstrass's Theory of Theta-Functions (Spring 1888).
9. Applications of the Theory of Elliptic Functions (Autumn 1888).
10. Weierstrass's Theory of Elliptic Functions (Autumn 1889).
11. Theory of Partial Differential Equations (Spring 1890).
12. Application of Analysis to Number Theory (Autumn 1890).

In 1884 she published a second paper on the propagation of light in crystals ([10] and [11]), but was much embarrassed when Vito Volterra pointed out a fundamental error in which (following Lamé) she had treated a certain multi-valued function as though it were single-valued. She wrote to Weierstrass, reproaching him for not having detected her mistake before publication. In his reply of 13 September 1884, Weierstrass wrote: "My dear friend, I'm all contrite under the burden of the reproaches which are contained in your letter", and excused himself by telling her of his illnesses and overwork. ([34], p.115)

In 1886 J.J. Sylvester (whom she described as "a youngster of 72" [28], p.705) wrote a sonnet in her honour, and published it in Nature (Fig.2).

MUSIC AND MATHEMATICS

YESTERDAY afternoon meeting at a friend's house a lady visitor to Oxford who was to sing that evening at one of the hebdomadal concerts in Balliol College, and the conversation happening to turn on the gifted mathematical lady Professor in the University of Stockholm, my thoughts shaped themselves, as I was walking home, into the following lines, which, if likely to interest any of your readers, I shall be happy to see appear in the world-wide-diffused columns of NATURE.

New College, November 15 J. J. SYLVESTER

SONNET

To a Young Lady about to sing at a Sunday Evening Concert in Balliol College

Fair maid! whose voice calls Music from the skies
Weaving amidst pale glimpses of the moon
Tones with fresh hues of glowing fancy strewn
And soft as dew that falls from pitying eyes—
Let from their virgin fount those accents rise
That bid sad Philomel suspend her tune,
Thinking the lark doth chant his lay too soon—
Whose else that trill which with her own note vies!
To her whose star shines bright o'er Maelar lake
And thee who beautifi'st glad Isis' shore
Grant! I one joint harmonious garland bind:
Thou canst with sounds our senses captive take—
She the true Muse, fond poets feigned of yore,
Strike Heaven's own lyre, Nature's o'erruling mind.

Nature, 9 December 1886, p.132. Reproduced by permission of the Editor of Nature. (Maelar Lake is on the outskirts of Stockholm.)

Figure 2

She had a wide circle of friends in many countries, consisting principally of scientists, writers and revolutionaries. Amongst mathematicians, her closest friends included Weierstrass, Chebyshev, Mittag-Leffler, Hermite and Kronecker. She translated two papers by Chebyshev from Russian into French, and published them in Acta Mathematica [2] and [3].

Her greatest mathematical triumph came in 1888, when the Paris Academy of Sciences proposed the subject of "the theory of the motion of a rigid body around a fixed point" for the Prix Bordin; a topic which had been proposed on previous occasions, without any prize then being awarded. Of the 15 entries received, one was judged so

meritorious that the judges increased the prize from 3000 to 5000 francs; and when the judges opened the sealed envelope containing the author's name, on 18 December 1888, they found that they had awarded the prize to Sof'ya. Six days later, at a special session of the Academy the prize was presented to Sof'ya, who became the toast of Paris. The Swedish Academy of Sciences awarded her a prize in 1889 for extensions to her Paris essay. ([13], [14], [15] and [31], ch.9)

Euler had solved the problem of the motion of a rigid body in free fall, or (equivalently) suspended at its centre of mass in a gravitational field. Lagrange had solved the very difficult problem of the motion of a rigid body in the case where it is suspended at a point such that the body's ellipsoid of inertia is symmetrical about an axis, and the centre of mass also lies on that axis of symmetry. Sof'ya considered all cases in which the coordinates of the rigid body can be expressed as power series of time, and she showed that, apart from the cases of Euler and Lagrange, all such cases consist of a body suspended at a point for which the body's ellipsoid of inertia is an oblate spheroid of revolution, with the equatorial radius twice the polar radius, and with the centre of mass lying in the equatorial plane. In a brilliant display of virtuosity, she constructed the coordinates explicitly as ultra-elliptic functions of time. (Actually, she had overlooked the possibility of the coordinates having an essential singularity at time zero, but that minor omission was rectified by A.M. Lyapunov [7] App.5)

In 1889 Chebyshev, together with Imshenetskiĭ and Bunyakovskiĭ, managed to get the St. Petersburg Academy of Sciences to amend its Charter so as to permit women to become members. The three of them then proposed Sof'ya for membership, and on the 20th November 1889 Chebyshev sent the following telegram to Sof'ya: "Our Academy of Sciences has just now elected you as a Corresponding member, having just permitted this innovation for which there has been no precedent

until now. I am very happy to see this fulfilment of one of my most impassioned and justified desires. Chebyshev". ([31] p.64 and [20] p.352). Weierstrass congratulated her on 5 February 1890, at having finally gained some recognition in Russia for her academic achievements. ([34] p.149)

Her last mathematical paper was a note on potential theory, published in 1891 [16].

Sof'ya engaged in much literary work from 1886 onwards. She published several articles in Swedish journals, including a noteworthy memoir of George Eliot. Her beloved sister Anna died in 1887, after Sof'ya (and also Dostoyevskiĭ's widow) had nursed her through a long and painful illness. Brooding over memories of their childhood, she commemorated Anna by her book of *Recollections of Childhood*, which was published first in a Swedish adaptation and then in a (censored) Russian version. It was immediately acclaimed as a classic account of a child's life in Russia, and it was subsequently translated into many languages. She also wrote poems, stories, plays and novels which were well-received, except by the Russian censor, who banned most of them. Her major novel was *A Nihilist Girl* [18], and she began writing a novel about Chernyshevskiĭ entitled *The Nihilist*. ([21] pp.157-181)

By 1888, Sof'ya was feeling rather frustrated with life in the small town of Stockholm, and was becoming increasingly homesick for Russia. ([33] pp.259-271). At Moscow University the chair of state law had been held by Maksim Maksimovich Kovalevskiĭ (no relation of Sof'ya's late husband Vladimir), but he had been dismissed on suspicion of holding radical views. (Those suspicions were justified.) Sof'ya was a member of a committee which invited him to Stockholm University in February 1888, to deliver a series of lectures on sociology. He dedicated the published lectures to her, and they

embarked on a tempestuous love affair which became the scandal of Europe. He proposed marriage, but she would have had to give up her post as Professor of Mathematics, and that she was not willing to do.

In February 1891, after an unhappy meeting with Maksim Maksimovich in Genoa she returned to Stockholm, where she succumbed to pneumonia and died on 10 February 1891. [33]

Her funeral was an occasion for widespread mourning, with tributes to her memory being presented by many friends and colleagues. Mittag-Leffler's brother distributed printed copies of a poem which he had written in honour of her life and achievement. ([22] p.176)

Verses written on Sof'ya Kovalevskaya, after her Death,
by F. Leffler,
translated from the Swedish by L. Waara.

Soul of fire, soul of thought,
has your air-ship weighed her anchor
now to ply the starry space
for ever, where your thought sublime
used to linger many a time,
and to where it soared aloft,
searching for the cause of Cosmos,
when in the starry evening
the ring of Saturn was seen gleaming
on the dark-blue vault?

Maybe now from higher spheres
analytic functions will
let you find an answer to
everlasting life's enigma?

The rays of light from high above
you used to view with searching eye
how they were in glass refracted.
How do now you see light flow?

From the bright celestial worlds
you often turned your gaze away
down to darkness centred here
on this miserable earth.
There also, in hopeful moments,
you could see the crystal surface
- made of Love - refract the light
and replace the power of darkness.

Soul of fire, soul of thought,
Love's anchor was the most secure.

* * * * * * * *

Thanks and farewell, then! - Not heavy
may the Swedish soil now cover
the young life which is delivered
to the haven of the tomb!
As long as Saturn's ring revolves
on its way amongst bright worlds,
and men live, your memory will
be hallowed amidst great souls.

The Russian Minister of the Interior, however, protested that far too much attention was being paid to "a woman who, when all is said and done, was nothing but a Nihilist!". ([20] p.532)

I wish to express my thanks to Paul Hafner for translating German and French material, and to Leonard Waara for translating F. Leffler's elegy on S.V.K. from Swedish to English.

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