Elements of Mathematics: from Euclid to Gödel by John Stillwell, 2016, Princeton University Press, 440 pp, ISBN: 978-0691171685, £29.95.

The inspiration for this book is Felix Klein's celebrated three-volume *Elementarmathematik vom höheren Standpunkte aus*, made available by Springer last year in a new English translation as *Elementary Mathematics from a Higher Standpoint*. Klein's audience was secondary school mathematics teachers: he offered them a panorama of the material they taught in the context of its historical development through to its place in the landscape of modern (1900s) mathematics. Stillwell's main goal is to revisit Klein's achievement from a twenty-first century perspective. In addition he has a philosophical aim: "to explain what 'elementary' means".

Klein's third volume is finally available in English for the first time. Stillwell has commented elsewhere on the lack of an earlier translation: "Evidently his subject matter [*Precision Mathematics and Approximation Mathematics*, in the new translation] was of lesser interest to English-speaking mathematicians of his time, and one doubts that it would be of much interest today, now that computers have completely revolutionized the practice of numerical and graphical approximation." Perhaps a more likely explanation is the early death in 1943 of Earle Raymond Hedrick, co-translator with Charles Noble of volumes 1 and 2. Besides, volume 3 concerns itself with applied mathematics, particularly geometrical applications of calculus. Its material is beyond the scope of Stillwell's present book but certainly deserves a similar treatment notwithstanding the computer revolution.

Computational mathematics constitutes the bulk of what in Stillwell's book would have been unrecognisable to Klein, who was, after all, still re-editing his books until he died in 1925. Combinatorics and probability and statistics are topics not dealt with per se by Klein, but only what was known up to the 1920s is covered by Stillwell. The treatments of algebra and vector geometry are correctly based in the work (recognised by Klein) of Grassmann who died in 1877; number theory is development as algebraic number theory up to circa 1900; calculus is given in more or less as the same manner as in Klein's volume 1.

Computation apart, Stillwell is not offering a new century's worth of mathematics (the subtitle of his book says as much). But of course his selection of topics and their presentation is thoroughly modern and is very well done. His intended reader, already having 'a good high school training' and having read and enjoyed everything in this book, would be equipped to read any semi-technical account of any part of current pure mathematics. In answer to the question 'what is elementary mathematics?' one might reasonably say 'what is covered in Stillwell's book'.

But Stillwell is hoping for a non-self-referential answer and in this I feel he is not successful. Tentative suggestions are scattered through the book: "Mathematics without infinity ... could be a candidate for 'elementary mathematics'"; "non-Euclidean geometry is more advanced than Euclidean ... since non-Euclidean geometry was discovered more than 2000 years after Euclid"; "...deep questions about set theory and infinity, which are *very* advanced mathematics." (Stillwell's emphasis). There is some coverage of Harvey Friedman's 'reverse mathematics' programme in which 'less elementary' theorems are those necessitating stronger axiom systems. To me this seems rather removed from the spirit of the rest of the book, and from the spirit of Klein's books: would it make sense to tell secondary mathematics teachers that they must limit their material to, say, what is provable from Zermelo–Fraenkel minus the Infinity axiom?

There is a related but more informal and flexible interpretation of the word 'elementary' which is how it is used in the phrase 'elementary number theory'. It means, I suppose, something like 'without importing heavy machinery'. Maybe we can only assemble isolated examples but they are exciting and intuitive. Stillwell would, on the evidence here, relate the stories compellingly. The elementary proof of the prime number theorem is the obvious candidate (alas analytic number theory is absent from the book) or there is Lovász's stunning 1978 proof of Kneser's conjecture in combinatorics using the Borsuk–Ulam theorem, matched only after nearly 25 years with an elementary proof, due to Jiřì Matoušek. Every mathematician will have their favourite examples. Quite relevant is the recent book of John W Dawson, *Why Prove it Again?* (Birkhäuser, 2015) which collects case studies, quite informally but very interestingly, addressing its title.

Still, this was Stillwell's call to make and we cannot complain too much: he has written a very fine book full of beautiful mathematics expertly described. I recommend it.

A closing remark: in 2008, to mark the centenary of Klein's *Elementarmathematik*, the production of a modern rewriting was initiated by the IMU through its International Commission on Mathematical Instruction (of which Klein was the founding president). No such book appears to be forthcoming, perhaps the idea was made redundant by the appearance of Stillwell's book. However the project has spawned a useful blog called blog.kleinproject.org which publishes mathematical 'vignettes' in the spirit of Klein's book.