

Chapter 1. Section 1

Welcome

This e-book comprises the weekly sets of notes that I prepared to accompany a course *Algorithmic Graph Theory* that I taught for a number of years at Queen Mary University of London. The course owes its structure to Bill Jackson and Peter Keevash who taught it before me. Also to Aiden Roy, who was supposed to be their successor, indeed had already started teaching it to a very appreciative cohort, only to be disinherited by a silly UK employment law ruling that UK students should not be allowed access to a Canadian graph theory expert. His were hard shoes to step into!

The course is largely a set of variations on the theme of greedily growing spanning trees. So minimum spanning tree and shortest path algorithms, rather than just finding themselves part of a disparate programmer's toolkit, are rather what enables pretty well everything else therein. The old dig that graph theory consists of a thousand definitions and only one theorem echoes as a reminder that, as in any relatively young theory, the plot can be harder to discern, particularly for those learning the trade. I was grateful, as I think were my students, to be offered a plot line that was memorable and easy to follow.

The plot running through these notes is, nevertheless, not what one might choose to follow today, some twenty years (at the time of writing) after the course was conceived. The algorithms on display belong to what is referred to affectionately as 'Hungarian graph theory', in homage to the tradition of the subject's first textbook, the 1936 *Theorie der endlichen und unendlichen Graphen* of Dénes König. Even as I taught the course, research in graph theory was swivelling rapidly towards the probabilistic approach (of which, ironically, Peter Keevash, is a stellar exponent). The theory of graph minors also, has resolved many problems that defeated 'Hungarian' methods and normally has algorithmic implications. I don't think a graph algorithms course today (and this was, after all, a level 3 course) could not mention expander graphs, Szemerédi's theorem, and tree width.

Still, it is with pleasure, even if it is nostalgic pleasure, that I archive these notes. At all events, many of the algorithms here solve iconic optimisation problems in a tried-and-tested and easy to implement way, doing service until or unless cutting-edge technology is required. The archive in question is my theorems website theoremoftheday.org, which allows me to link to appropriate theorem pages and their accompanying notes. A case in point: my notes page for the Max-flow min-cut theorem records that "There are probabilistic algorithms for Max-flow which are much faster than Ford-Fulkerson", citing recent arxiv and blog postings, the latter eliciting debate from experts in the field.

There again, it has become more desirable that mathematics with immediate real-world application should be taught with the real-world providing examples of, or even the *raison-d'être* for, the mathematics. Again I may take refuge behind my theorems website: "There is a nice real-life application [of Euler tours] to snow ploughing described here..."

Most importantly, though, these notes assemble a string of classic showpieces of discrete mathematics: algorithms and theorems that are, and will remain, an inspiration for practitioners in mathematics, information technology and, who knows, artificial intelligence!