Graph Algorithms

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Chapter 10. Section 1

Euler Tours and Eulerian Graphs

Euler tours: new problem whose solution is not based on tree-growing!

leading to

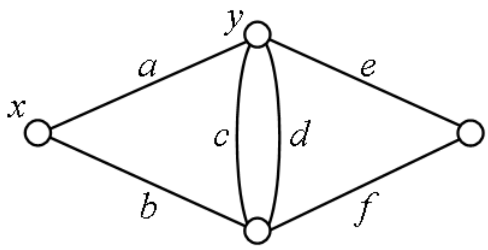
Chinese Postman: weighted variant of Euler tour problem solved by

Euler tour + Dijkstra + Matchings.

Recall some essential terminology:

Multiple edges: distinct edges having the same two endpoints;
Consecutive edges: $e = xy$ and $e' = x'y'$ are **consecutive** if $y = x'$;
Walk: sequence of consecutive edges, the number of edges in the sequence being the **length** of the walk;
Connected graph: graph in which any two vertices have a walk joining them;
Closed walk: walk whose first and last vertices are identical;
Trail: walk with no repeated edges;
Closed trail: trail whose first and last vertices are identical;
Path: trail with no repeated vertices, except perhaps first and last, in which case we have:
Cycle: path whose first and last vertices are identical.

Example: we determine the walks of various types and lengths (length = no. of edges) joining x and y in the graph:



length	walks	trails	paths
2	bc, bd	bc, bd	bc, bd
3	bfe, acd, adc, acc, add, aee, bba, aaa	bfe, acd, adc	bfe
4	$acfe$, $adfe$, $aefc$, $aefd$, $aabc$, and 19 others	$acfe$, $adfe$, $aefc$, $aefd$	none

Definition In a graph G an **Euler tour** is a closed trail containing every edge. A graph is called **Eulerian** if it admits (i.e. 'has') an Euler tour.

Note: these terms honour the Swiss mathematician Leonhard Euler (pronounced '**Oil**-er') (1707–1783).

Example: we give some Euler tours of the above graph:

(1) $a\,c\,d\,e\,f\,b$

(2) $b\,f\,e\,d\,c\,a$ (which is (1) reversed)
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(3) $a\,c\,f\,e\,d\,b$
(which is (1) with the cycle def reversed)

(4) $c\,d\,e\,f\,b\,a$
(which is (1) but 'cycled round' to start at vertex y)