Graph Algorithms

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Chapter 10. Section 3

The Recursive Euler Tour Algorithm

We proved that even degrees in a connected graph is sufficient to guarantee an Euler tour (the Euler-Hierholzer Theorem). Our inductive proof directly implies an algorithm to construct a tour.

Indeed, any inductive proof implies an algorithm: the size N case is constructed in the **induction step** from the size $N-1$ case. This in turn is then constructed from the $(N-1)-1$ case. The reduction continues until we reach the **base case** which is constructed (proved) separately.

If we write the algorithm down explicitly it is naturally expressed using **recursion**: the algorithm is defined in terms of itself, but applied to a smaller input; in order to terminate, a smallest possible input (base case) is specified, for which the output is specified separately.

Here is a recursive algorithm for constructing an Euler tour:

Recursive_Euler_Tour(G)
 # Input: connected graph G in which all vertices have even degrees
  if $E(G)=\emptyset$ **then return**(**( )**) **fi**; *# the base case: G is a single vertex;* **( )** *is the empty tour.*
  W := a walk in G which forms a cycle, say, C;
  G' := $G\backslash C$, with connected components G_i, $k\geq 1$; *# $G\backslash C$ is G with the edges of C deleted.*
  for i := 1 to k **do**
    S_i := Recursive_Euler_Tour(G_i); *# the recursive step.*
  od;
  x_i := a vertex in which G_i intersects C in G, $i=1,\ldots,k$;
  S := C with S_i (cycled to begin and end at x_i) inserted at x_i;
  return(S)

Example: in the graph G on the right the walk $W=a\,e\,g\,j\,f$ forms the cycle $C=g\,j\,f$

Deleting the edges of C gives a new graph ...

... with two connected components.

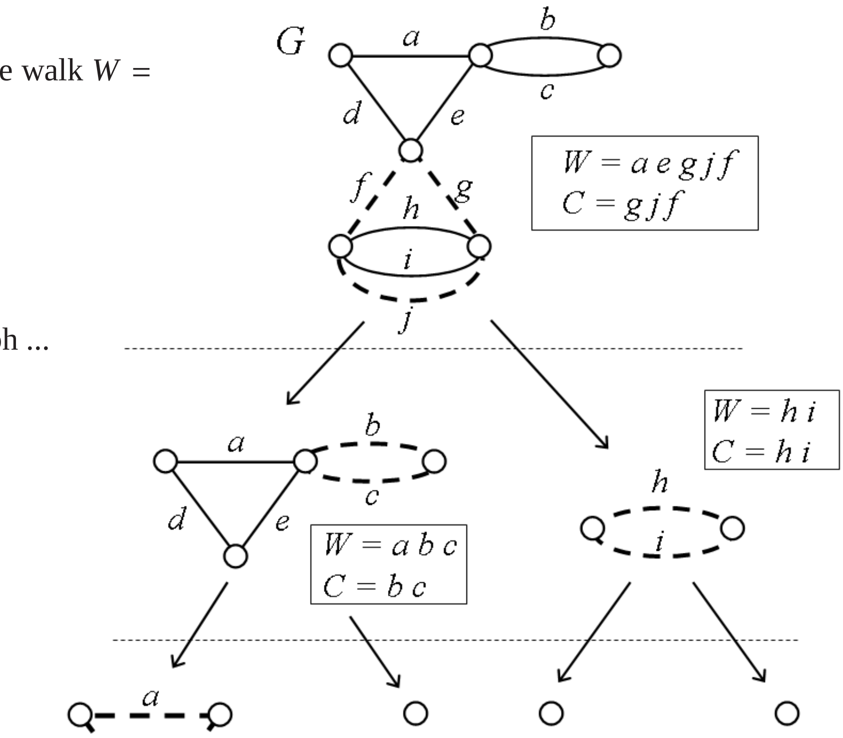
Each gives a new cycle.

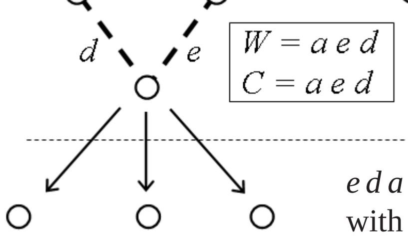
Deleting these gives two new graphs ..

... each with two components

Three of this total of four components are trivial, with empty tours ( ).
The other has a new cycle.
Deleting this gives ...

... three trivial graphs, with empty tours ( ).





Now the ( ) are incorporated into $a\,e\,d$. This is cycled round to incorporate into $b\,c$ to give $e\,d\,a\,b\,c$. This is cycled to incorporate, with $h\,i$, into $g\,j\,f$, giving a complete Euler tour: $d\,a\,b\,c\,e\,g\,h\,i\,j\,f$