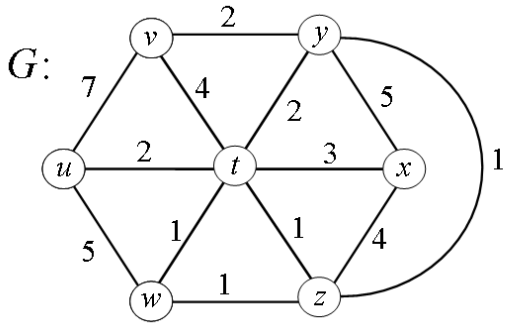
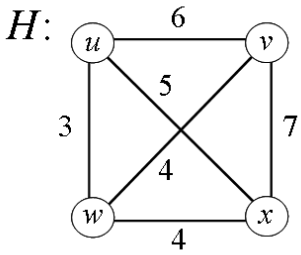
Chapter 11. Section 2

The Chinese Postman Problem

We now move to a very general problem: our connected graph G has vertices of arbitrary degree and edges weighted over $\mathbb{R}^{\geq 0}$. We wish to find a closed walk of minimum total weight which passes every edge. This is called the **Chinese Postman Problem**[1]

Consider the graph G, below left:

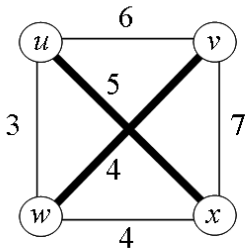


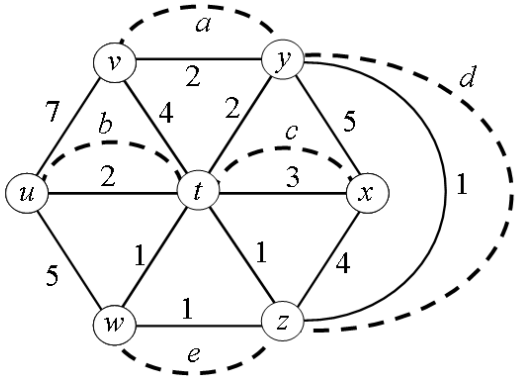


We want to solve the Chinese Postman Problem with a closed walk at vertex t. How do we do it?

The algorithm is:

1. Construct a graph H whose vertices are the odd-degree vertices of G and with every pair of vertices adjacent (a **complete graph**);
2. Apply Dijkstra's algorithm at each odd-degree vertex of G to find the minimum-weight paths from this vertex to each other odd-degree vertex. Put these minimum weights on the edges of H. For our example graph, the H is given, above right;
3. Find a minimum-weight perfect matching in H: that is, among the perfect matchings of H, choose one whose edges have the minimum total weight; such a matching shown below left;
4. For each pair of vertices of H joined by a matching edge, double the edges between the corresponding vertices in G along a minimum-weight path between these two vertices: see below right;
5. Find an Euler tour in the new version of G;
6. The required closed walk in G is the Euler tour with doubled edges representing edges of G which are used twice.





Here is a solution for our example graph:

Euler tour in new graph: *tx* ***c*** *tz zx xy yt tu* ***b*** *tv vy* ***d*** *zy* ***a*** *vu uw wz* ***e*** *wt*
Closed walk passing every edge in G: *tx* ***xt*** *tz zx xy yt tu* ***ut*** *tv vy* ***yz*** *zy* ***yv*** *vu uw wz* ***zw*** *wt*

[1]It was first discussed by Guan Mei-Gu (sometimes transliterated as Kwan Mei-Ko) in a paper published in China in 1962. Alan Goldman suggested the problem, and the name, to Jack Edmonds while they were both at the National Bureau of Standards in the 1960s. Edmonds' joint paper with Ellis Johnson, reducing the problem to a matching problem, appeared in 1973. Guan continued to work on the problem at the Shandong Normal University and invented a much harder variant called the Windy Postman Problem. He is now, I believe, at Fudan University in Shanghai.