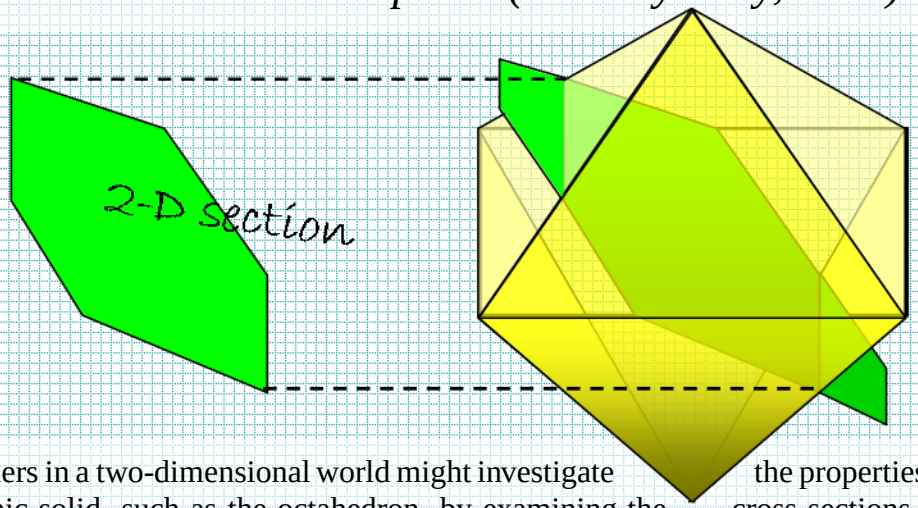
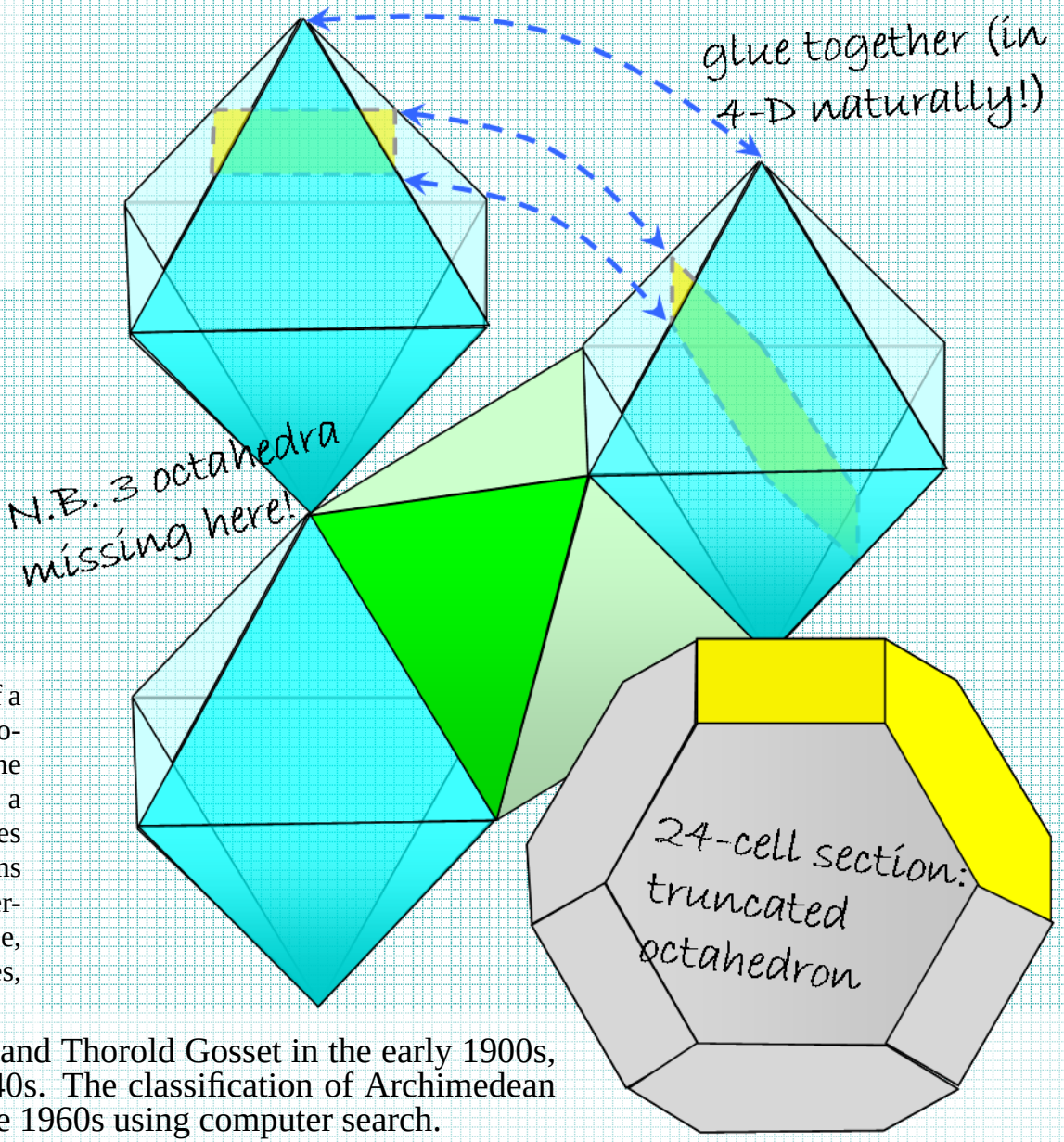
THEOREM OF THE DAY

The Classification of Archimedean 4-Polytopes *The Archimedean polytopes in four dimensions are:*

1. *the polytopes of Boole Stott–Wythoff type;*
2. *the prismatic polytopes;*
3. *Cartesian products of regular polygons;*
4. *Thorold Gosset's snub-24-cell (1900);*
5. *the Grand Antiprism (Conway–Guy, 1965)*





Dwellers in a two-dimensional world might investigate the properties of a Platonic solid, such as the octahedron, by examining the cross-sections produced when a plane intersects it. Above, a plane intersection parallel with one of the faces produces a hexagonal section; a different intersection might reveal a square or a rectangle. Archimedean four-dimensional polytopes consist of copies of prisms and Archimedean solids meeting at vertices in identical configurations (more precisely, the symmetry group of the polytope acts transitively on its vertices). A drawing by Boole Stott is adapted, right, to show how a regular polytope, the 24-cell, consisting of 24 octahedra with six meeting at each of its at 24 vertices, may be 'unfolded' so that 3-dimensional sections through it may be visualised.

Following pioneering work by Alicia Boole Stott, Willem Wythoff and Thorold Gosset in the early 1900s, work on polytopes was systematised by H.S.M. Coxeter in the 1940s. The classification of Archimedean 4-polytopes was completed by J.H. Conway and Michael Guy in the 1960s using computer search.

Web link: johncarlosbaez.wordpress.com/2012/05/21/. Fine descriptions of Boole Stott's work are www.ams.org/featurecolumn/archive/boole.html by Tony Phillips and www.sciencedirect.com/science/article/pii/S0315086007000973 by Irene Polo-Blanco.

Further reading: *The Symmetries of Things* by John H. Conway, Heidi Burgiel and Chaim Goodman-Strauss, A.K. Peters, 2008, Chapter 26.