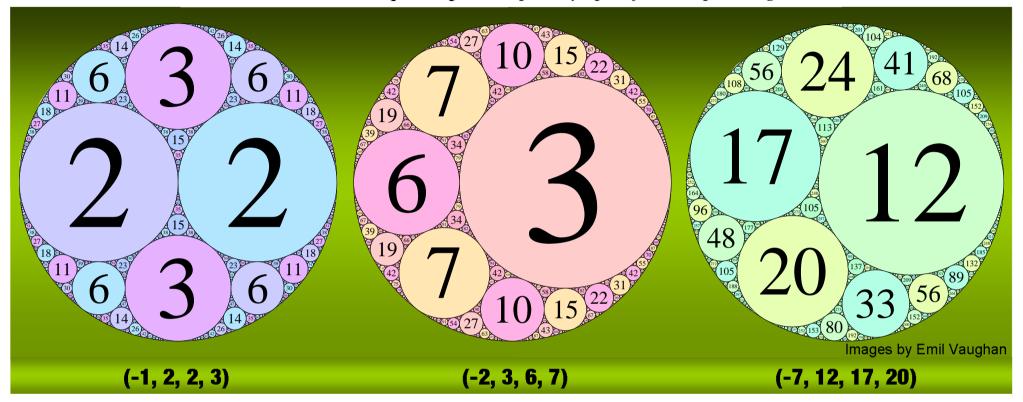
THEOREM OF THE DAY



A Theorem on Apollonian Circle Packings For every integral Apollonian circle packing there is a unique 'minimal' quadruple of integer curvatures, (a, b, c, d), satisfying $a \le 0 \le b \le c \le d$, a+b+c+d > 0and $a + b + c \ge d$. This so-called root quadruple completely specifies the packing.





A Descartes configuration consists of four mutually tangent circles. Above right, for example, is a circle of radius 1/7 containing circles of radius 1/12, 1/17 and 1/20, each of which has a point of contact with the other three. The integers labelling the circles are the *curvatures* (the reciprocals of the radii) and in the root quadruple of curvatures, (-7, 12, 17, 20), the enclosing circle of radius 1/7 is determined to have negative curvature so that all four circles have disjoint interiors. Any such configuration specifies four more tangent circles — above right, these have curvatures 24, 33, 48, and 105, producing four new configurations (-7, 12, 17, 24), (-7, 12, 20, 33), (-7, 17, 20, 48) and (12, 17, 20, 105). Repeating this process produces a system of infinitely packed circles: an Apollonian circle packing. If our initial configuration is integral, as in each of the above examples (which are drawn to different scales), then we will get an *integral* packing with every curvature an integer.

This theorem comes from a series of four pivotal papers by the AT&T team of Ronald Graham, Jeffrey Lagarias, Colin Mallows and Allan Wilks, together with Catherine Yan of Texas A&M University. They further show that all integral Apollonian circle packings may be derived from root quadruples which, like those depicted above, have entries whose gcd is 1.

Web link: www.ams.org/featurecolumn/archive/kissing.html; more advanced: www.maa.org/programs/maa-awards/writing-awards/integral-apollonian-packings. A good guide to creating your own circle packings is Cliff Reiter's article at archive.vector.org.uk/22/4.



