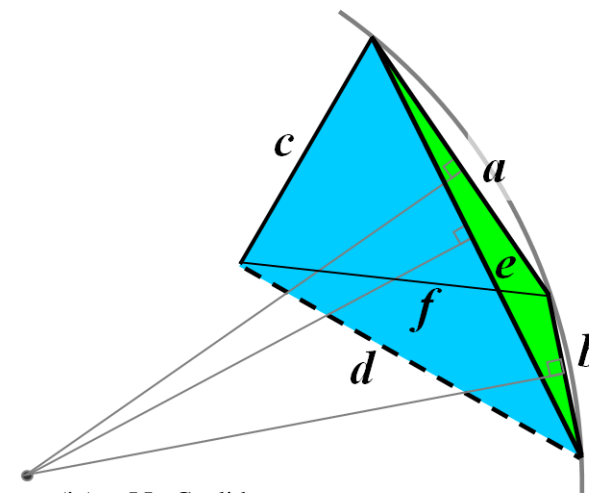
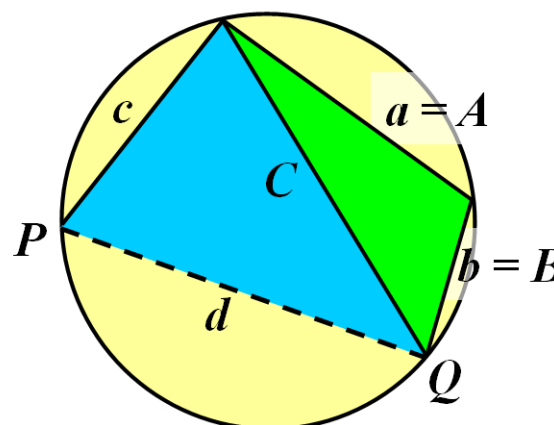
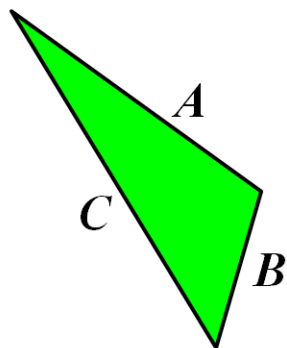
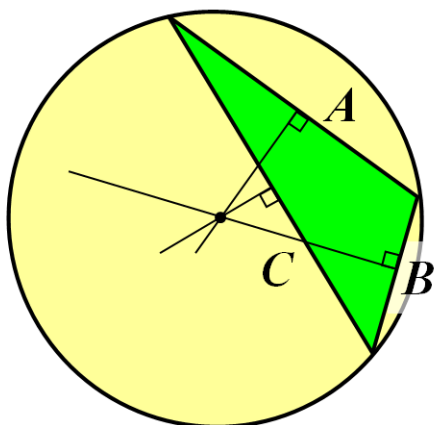




THEOREM OF THE DAY

Brahmagupta's Formula *The area K of a cyclic quadrilateral with side lengths a, b, c, d and semiperimeter $s = (a + b + c + d)/2$ is given by*

$$K = \sqrt{(s - a)(s - b)(s - c)(s - d)}.$$



(i) Euclid of Alexandria
Elements, Book IV, (c. 300 BC)
Circumcircling the triangle

(ii) Heron of Alexandria
Metrica, (1st century AD)
Sides \rightarrow area (triangle)

(iii) Brahmagupta
The Brahmasphutasiddhanta, (628 AD)
Sides \rightarrow area (cyclic quadrilateral)

(iv) J.L. Coolidge
Amer. Math. Monthly, (1939)
Sides \rightarrow area (any quadrilateral)

(i) The familiar formula for triangular area ($1/2 \times \text{base} \times \text{height}$) was known to Greek mathematicians for at least three hundred years before Euclid catalogued all of known geometry in his *Elements*, including (Book IV, Proposition 5) the construction of the unique *circumcircle* which passes through the vertices of a triangle, by intersecting the perpendicular bisectors of its sides.

(ii) Three hundred years after *that* came the famous **Heron's formula**: *the area K of a triangle with sides A, B, C and semiperimeter $s = (A + B + C)/2$ is given by $K = \sqrt{(s - A)(s - B)(s - C)s}$.*

(iii) Although Greek mathematics was apparently unknown to medieval Indian (as opposed to Islamic) scholars, Brahmagupta effectively put Heron's triangle back into the circle: take two non-overlapping circumscribed triangles sharing a common edge (C in the picture); the result is a *cyclic quadrilateral*, one whose vertices all lie on a circle. And now the area of the quadrilateral replaces the final s in Heron's formula by $s - d$. If point P is allowed to approach point Q then d becomes zero and c becomes C , recovering Heron.

(iv) Another thirteen hundred years pass and the circumscribing circle is removed once more in American mathematician Julian Lowell Coolidge's **quadrilateral area formula**: *the area of an arbitrary convex quadrilateral with sides a, b, c, d , a opposite to d , with diagonals e, f , and with semiperimeter s , is given by $K = \sqrt{(s - a)(s - b)(s - c)(s - d) - \frac{1}{4}(ad + bc + ef)(ad + bc - ef)}$. This generalises Brahmagupta by virtue of another classic of antiquity, **Ptolemy's Theorem**: *quadrilateral a, b, c, d , a opposite to d , with diagonals e, f , is cyclic if and only if $ad + bc = ef$.**

Brahmagupta (598–670) was the first mathematician to treat zero as a number in its own right.

Web link: www.mathpages.com/home/kmath196/kmath196.htm

Further reading: *Journey Through Genius* by William Dunham, John Wiley & Sons, 1990.

