**Theorem of the Day**

Kepler’s Conjecture Any packing of three-dimensional Euclidean space with equal-radius spheres has density bounded by $\tau \sqrt{2}/12 \approx 0.74$.

The face-centred cubic packing aligns the spheres on a three-dimensional square grid or lattice; in hexagonal close packing no three layers align sphere centres but there is still regular structure and both packings achieve the density bound of Kepler’s conjecture, that is, the total empty space between balls totals $1 - \tau \sqrt{2}/12$, or a little less than 26%. But some non-regular arrangements do better for partial packings of three-space; ruling out such irregular packings makes Kepler’s conjecture particularly difficult to prove.

**Historical notes:**
- 1611: Johannes Kepler states conjecture in response to question of Thomas Harriot.
- 1890: Axel Thue proves the 2D analogue optimal circle packing has density $\tau \sqrt{3}/12$.
- 1900: Hilbert makes Kepler’s Conjecture one of his challenges for the 20th century part of his ‘18th problem’.
- 1953: Lázló Fejes Tóth reduces proof of Kepler’s conjecture to large but well-defined set of calculations.
- 1998: Thomas Hales (with grad. student Samuel P. Ferguson) announces computer-aided solution to these calculations.
- 2003: Hales launches FlySpeck project to verify computer proof using automated theorem-checkers (non-computer part of proof now generally accepted).
- 2012: Hales’ Dense Sphere Packings provides accessible, self-contained version of his proof, by now unassailable.

**Web link:** click on Cannonballs and Honeycombs at www.ams.org/notices/200004/index.html.

**Further reading:** Kepler’s Conjecture by George Szpiro, John Wiley & Sons, 2003; and Dense Sphere Packings by Thomas C. Hales, Cambridge University Press, 2012.