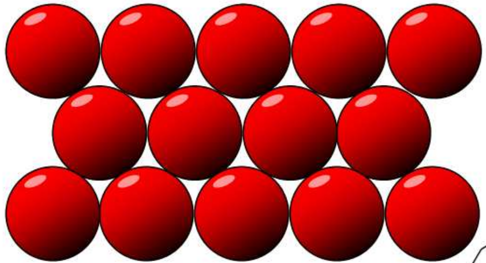
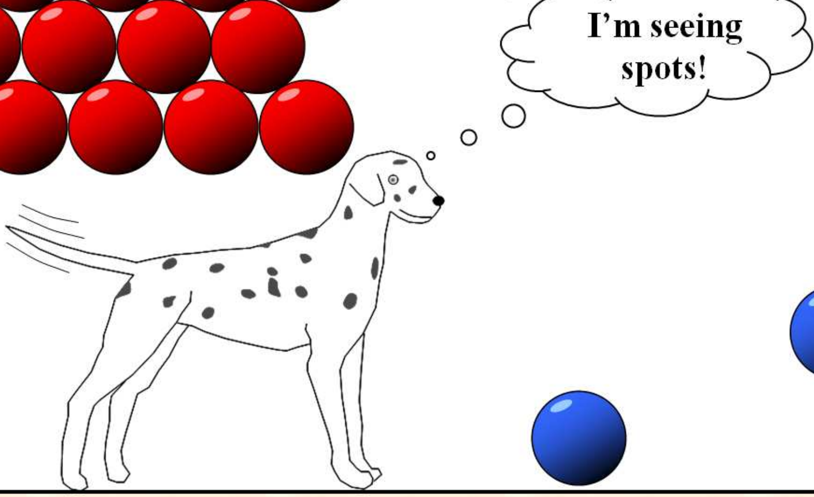
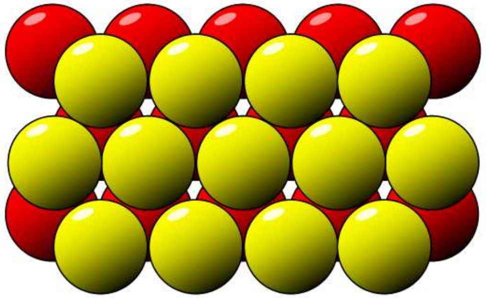
THEOREM OF THE DAY

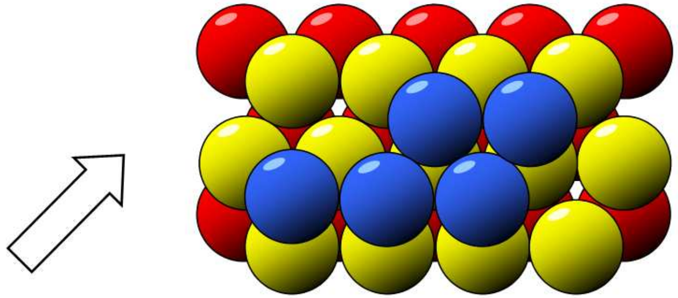
Kepler's Conjecture *Any packing of three-dimensional Euclidean space with equal-radius spheres has density bounded by $\tau\sqrt{2}/12 \approx 0.74$.*

The *face-centred cubic packing* aligns the spheres on a three-dimensional square grid or *lattice*; in *hexagonal close packing* no three layers align sphere centres but there is still regular structure and both packings achieve the density bound of Kepler's conjecture, that is, the total empty space between balls totals $1 - \tau\sqrt{2}/12$, or a little less than 26%. But some non-regular arrangements do better for partial packings of three-space; ruling out such irregular packings makes Kepler's conjecture particularly difficult to prove.



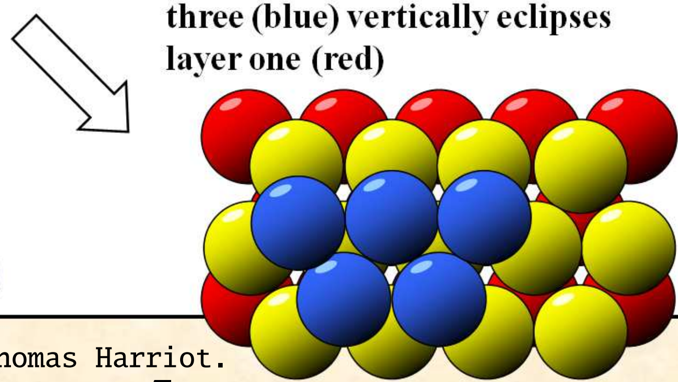


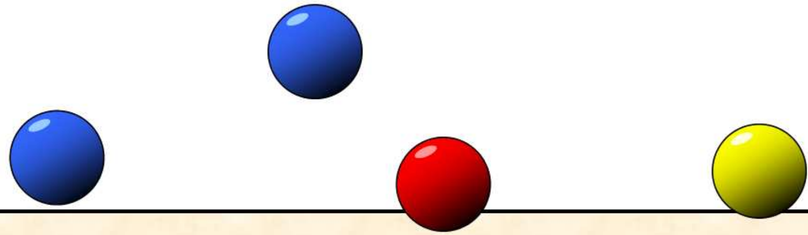




Face-centred cubic packing: layer three (blue) offset vertically against layer one (red)

Hexagonal close packing: layer three (blue) vertically eclipses layer one (red)





Historical notes:

1611: Johannes Kepler states conjecture in response to question of Thomas Harriot.
1890: Axel Thue proves the 2D analogue optimal circle packing has density $\tau\sqrt{3}/12$.
1900: Hilbert makes Keplers Conjecture one of his challenges for the 20th century part of his ‘18th problem.
1953: Lszl Fejes Tth reduces proof of conjecture to large but well-defined set of calculations.
1998: Thomas Hales (with grad. student Samuel P. Ferguson) announces computer-aided solution to these calculations.
2003: Hales launches FlySpeck project to verify computer proof using automated theorem-checkers (non-computer part of proof now generally accepted).
2012: Hales Dense Sphere Packings provides accessible, self-contained version of his proof, by now unassailable.
2014: Flyspeck completes on August 14.

Web link: Hales's 2000 overview for the AMS is the best introduction: click on *Cannonballs and Honeycombs* at www.ams.org/notices/200004.

Further reading: *Kepler's Conjecture* by George Szpiro, John Wiley & Sons, 2003; and Dense Sphere Packings by Thomas C. Hales, Cambridge University Press, 2012.