Miquel’s Triangle Theorem  Let $A$, $B$ and $C$ be the vertices of a triangle and $a$, $b$ and $c$ be points chosen on sides $CB$, $AC$ and $AB$, respectively. Then the circles defined by $bAc$, $cBa$ and $aCb$ have a common point of intersection. Moreover, if $a$, $b$ and $c$ are chosen to be collinear then this point lies on the circle defined by $A$, $B$ and $C$.

The above picture (created using David Joyces wonderful Geometry Applet package) shows Miquel’s Theorem in action. Changing the size or shape of triangle $ABC$, or moving any of the side points $a$, $b$ or $c$, will move but not destroy the point of intersection of the three red circles. The magenta triangle $abc$ reduces to a line when its vertices are collinear, as on the right, and at this point we find the red circles intersect in a point on the green circle on $ABC$.

Auguste Miquel was a French mathematician active in the mid-nineteenth century. The point of intersection of the circles in this theorem is known as the ‘Miquel Point’.

Web link: kskedlaya.org/geometryunbound/, see section 1.2 of “notes from August 1999” (under “Non-GFDL”).