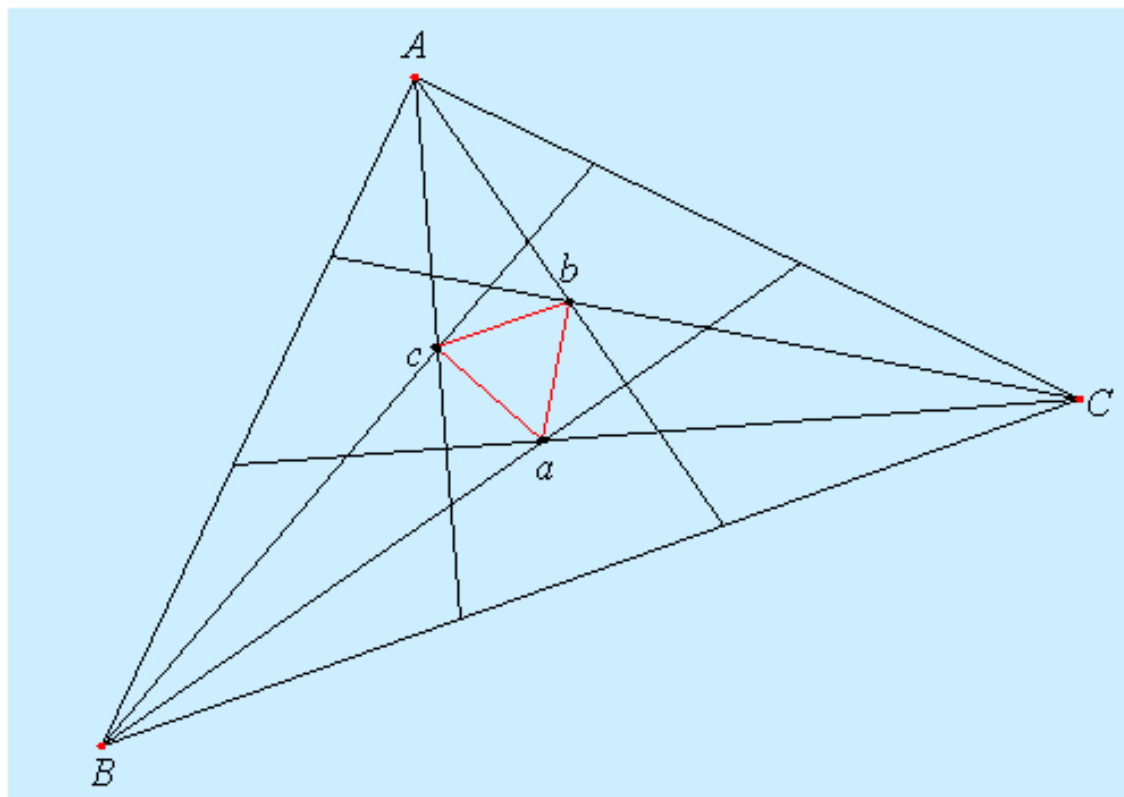




# THEOREM OF THE DAY

**Morley's Miracle** Let  $A, B, C$  be the vertices of a triangle. Let the angle  $BAC$  be trisected by lines  $A_B$  and  $A_C$ , in that order; similarly let  $C_A$  and  $C_B$  trisect angle  $ACB$ , and let  $B_C$  and  $B_A$  trisect angle  $CBA$ . Then the points of intersection,  $c = A_B \cap B_A$ ,  $a = B_C \cap C_B$ , and  $b = C_A \cap A_C$ , form the vertices of an equilateral triangle.



Trisecting angles is one of the famous 'impossibilities' of the Greek geometers, who did not have available to them David Joyce's wonderful Geometry Applet package: [aleph0.clarku.edu/~djoyce/java/Geometry/Geometry.html](http://aleph0.clarku.edu/~djoyce/java/Geometry/Geometry.html), used to create the image above. (Which is inanimate, unfortunately; the web link below has animations as well as at least 25 different proofs!)

Frank Morley (1860–1937) emigrated from England to Pennsylvania in 1887 to teach mathematics at the Quaker college at Haverford and discovered his theorem in 1899, perhaps more miraculous for having never been discovered before than for being surprising or powerful. Morley also excelled at chess, once beating world champion Lasker (also a mathematician). His son, Frank V. Morley, returned to England and became a director, with Geoffrey Faber and T.S. Eliot, of Faber & Gwyer, later Faber & Faber ("Morley, Faber and Eliot would sometimes communicate in exchanges of light verse." John Mullan writing in *The Guardian*, September 25, 2004)

**Web link:** [www.cut-the-knot.org/triangle/Morley/](http://www.cut-the-knot.org/triangle/Morley/)

**Further reading:** *Complex Numbers from A to ...Z*, by Titu Andreescu and Dorin Andrica, Birkhauser Boston, 2005, chapter 4, section 13.

