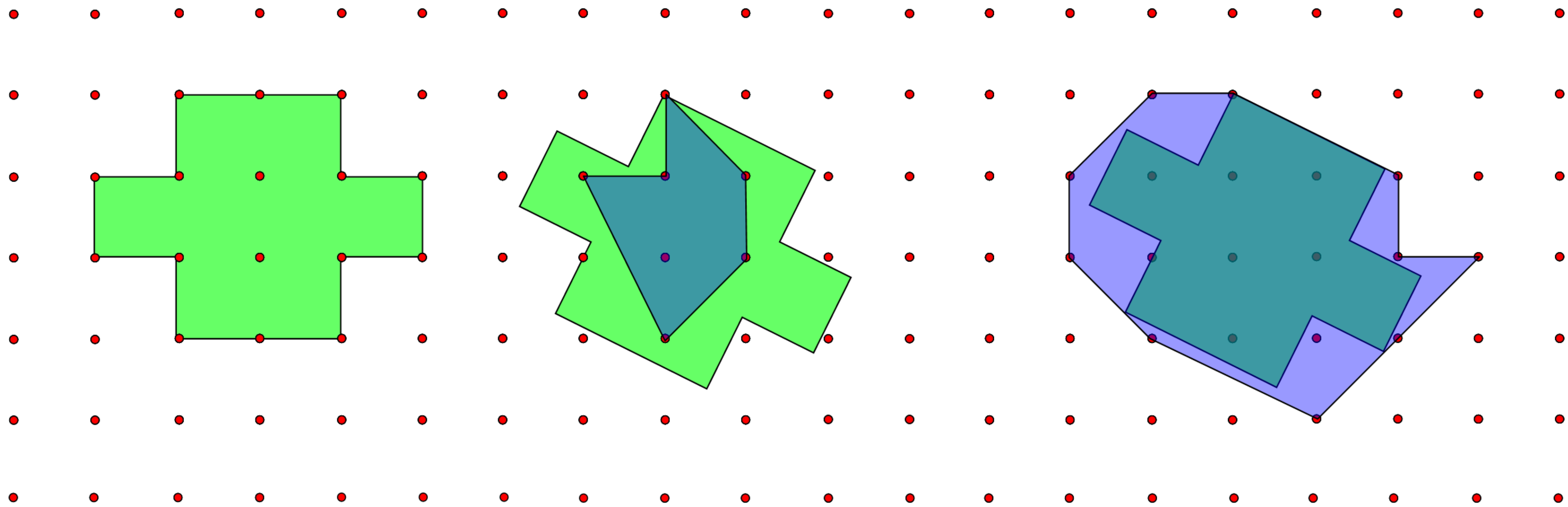
THEOREM OF THE DAY

Pick's Theorem *Let P be a simple polygon (i.e. containing no holes or separate pieces) whose vertices lie on the points of a rectangular lattice. Suppose that I lattice points are located in the interior of P and B lattices points lie on the boundary of P. Then the area of P is given by*

$$K = I + B/2 - 1.$$



The polygon on the left here has area $K = 2 + 14/2 - 1 = 8$ since it has $I = 2$ interior lattice points and $B = 14$ boundary points. In the centre, the same polygon has been rotated, whereby its area is clearly preserved. However the theorem does not apply because some vertices do not lie on lattice points; now $I = 6$ but B is only 2 and the area is underestimated to be $6 + 2/2 - 1 = 6$. However, we can estimate the correct area: the inside polygon has area $1 + 6/2 - 1 = 3$; on the right the outside polygon has area $8 + 10/2 - 1 = 12$. A reasonable estimate for the rotated polygon's area is the mean of the inside and outside areas: $(3 + 12)/2 = 7.5$.

Georg Alexander Pick (1859–1942) was an Austrian mathematician and physicist who published very widely in mathematics and was a friend of Einstein during the latter's time in Prague 1911–1913. He was sent to the Nazi death camp at Theresienstadt in 1942, aged 82, and died two weeks later. He published this elegant theorem in 1899.

Web link: www.cut-the-knot.org/ctk/Pick.shtml.

Further reading: *Mathematical Snapshots* by H. Steinhaus, Dover Publications, 2000 (the first, 1969, edition made Pick's Theorem famous).