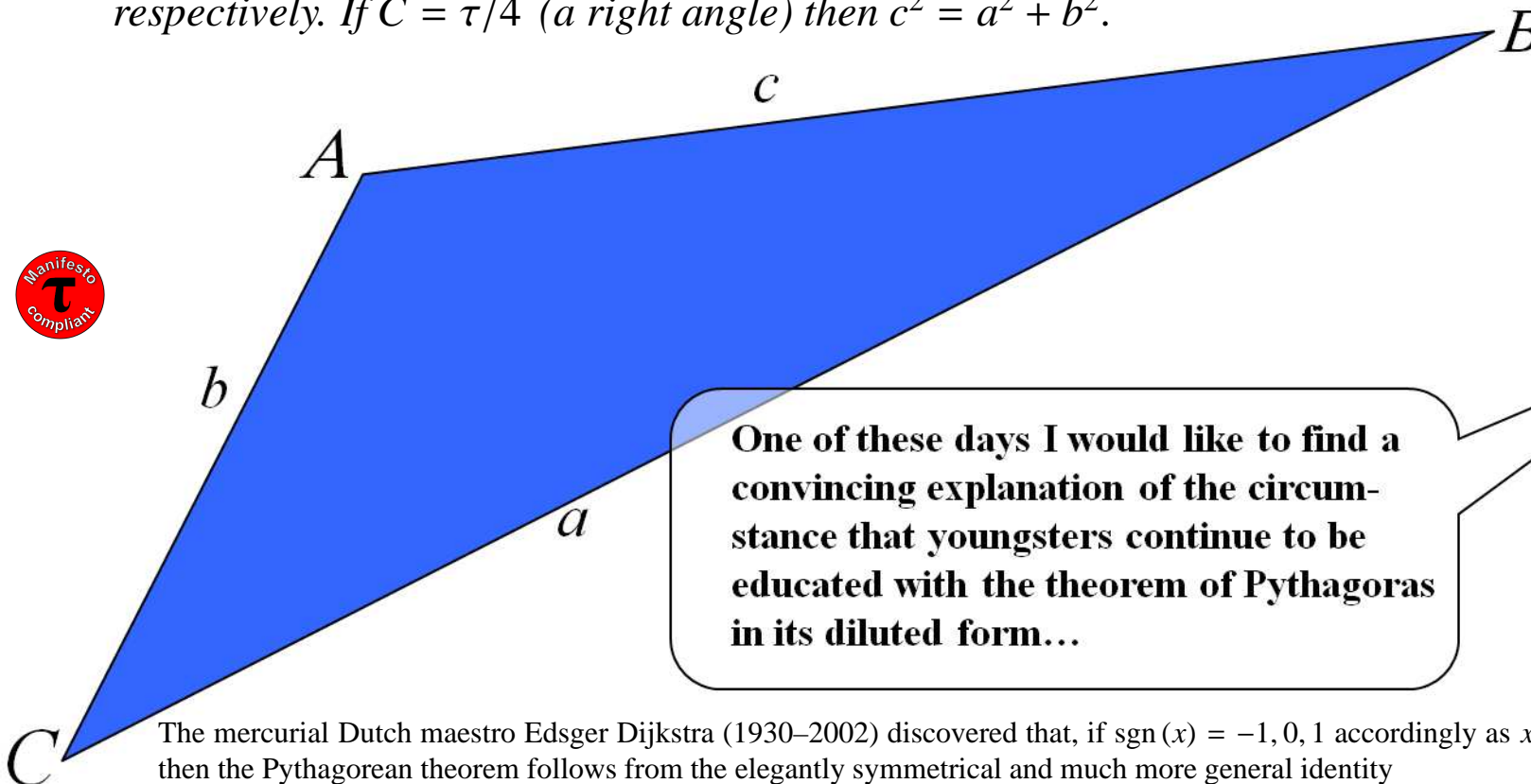




# THEOREM OF THE DAY

**The Pythagorean Theorem** Consider a triangle with angles  $A$ ,  $B$  and  $C$  and opposite sides  $a$ ,  $b$  and  $c$ , respectively. If  $C = \tau/4$  (a right angle) then  $c^2 = a^2 + b^2$ .



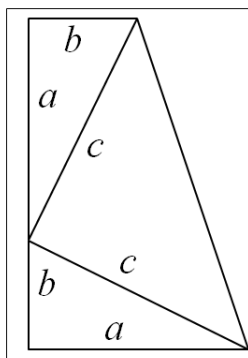
One of these days I would like to find a convincing explanation of the circumstance that youngsters continue to be educated with the theorem of Pythagoras in its diluted form...



The mercurial Dutch maestro Edsger Dijkstra (1930–2002) discovered that, if  $\text{sgn}(x) = -1, 0, 1$  accordingly as  $x$  is negative, zero or positive, then the Pythagorean theorem follows from the elegantly symmetrical and much more general identity

$$\text{sgn}(A + B - C) = \text{sgn}(a^2 + b^2 - c^2).$$

It is striking that a logical relationship between sides and angles can produce, when one angle is  $\tau/4$  radians (i.e.  $90^\circ$ ), a method for calculating one side from the other two.



**Proof:** area of trapezium = half (base + top)  $\times$  height =  $\frac{1}{2}(a + b) \times (a + b)$  = sum of areas of triangles =  $\frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$ . So  $c^2 = (a + b)^2 - 2ab = a^2 + b^2$ .

Sometimes known as ‘Pythagoras’s Theorem’, this is more accurately called ‘Pythagorean’ since it was almost certainly known before Pythagoras (569–475 BC). The Pythagoreans believed that the world could be described by the rational numbers which is ironic since their theorem produces triangles of sides such as  $\sqrt{1^2 + 1^2} = \sqrt{2}$  which is not a rational number.

**Web link:** Thony Christie is very informative on the history: [thonyc.wordpress.com/2014/04/16/](http://thonyc.wordpress.com/2014/04/16/). A pdf facsimile of Dijkstra’s original hand-written note on Pythagoras is archived as **Document 975** at [www.cs.utexas.edu/users/EWD/index09xx.html](http://www.cs.utexas.edu/users/EWD/index09xx.html). The proof given on the left is no. 5 (of over a hundred!) at [www.cut-the-knot.org/pythagoras/](http://www.cut-the-knot.org/pythagoras/).

**Further reading:** *The Pythagorean Theorem: a 4000-Year History* by Eli Maor, Princeton University Press, 2007.

