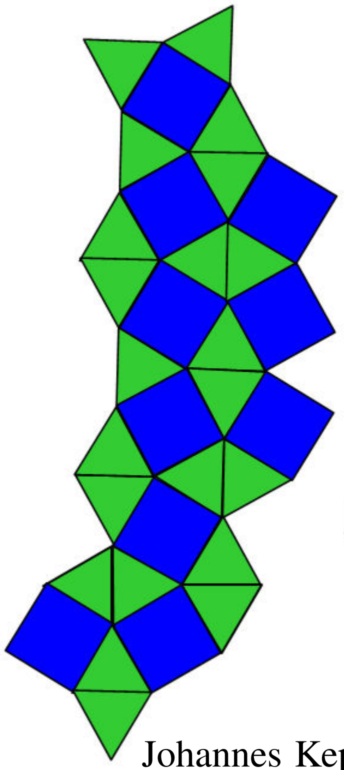
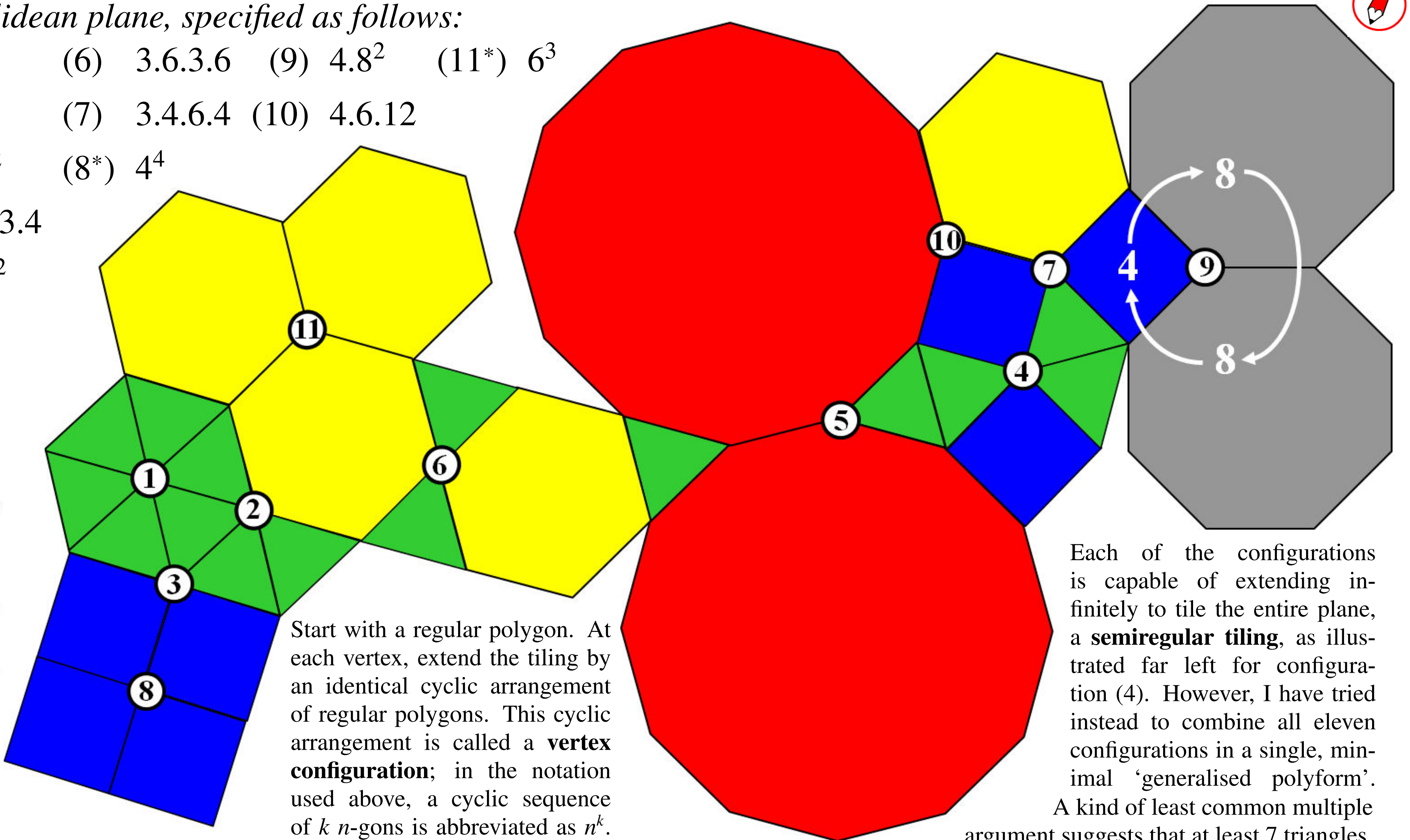
THEOREM OF THE DAY

The Classification of Semiregular Tilings *There are eight semiregular tilings and three regular tilings of the Euclidean plane, specified as follows:*

(1*)	3^6	(6)	3.6.3.6	(9)	4.8^2	(11*)	6^3
(2)	$3^4.6$	(7)	3.4.6.4	(10)	4.6.12		
(3)	$3^3.4^2$	(8*)	4^4				
(4)	$3^2.4.3.4$						
(5)	3.12^2						

* *regular*





Start with a regular polygon. At each vertex, extend the tiling by an identical cyclic arrangement of regular polygons. This cyclic arrangement is called a **vertex configuration**; in the notation used above, a cyclic sequence of k n-gons is abbreviated as n^k. E.g., the configuration at vertex 9 above is 4, 8, 8 = 4.8^2.

Each of the configurations is capable of extending infinitely to tile the entire plane, a **semiregular tiling**, as illustrated far left for configuration (4). However, I have tried instead to combine all eleven configurations in a single, minimal 'generalised polyform'. A kind of least common multiple argument suggests that at least 7 triangles, 5 squares, 4 hexagons, 2 octagons and 2 dodecagons are needed, a total of 20 polygons. But I have used 31 — perhaps you can do better?

Johannes Kepler classified the regular (configurations 1, 8 and 11) and semiregular tilings (also known as *Archimedean*) in chapter 2 of his 1619 *Harmonices Mundi* (in which also appears his third law of planetary motion). This classification was rediscovered independently by Paul Robin (in 1887), Duncan Sommerville (in 1905) and Alfredo Andreini (in 1907).

Web link: gruze.org/tilings/

Further reading: *Geometry: Plane and Fancy* by David A. Singer, Springer, 1998, Chapter 2.