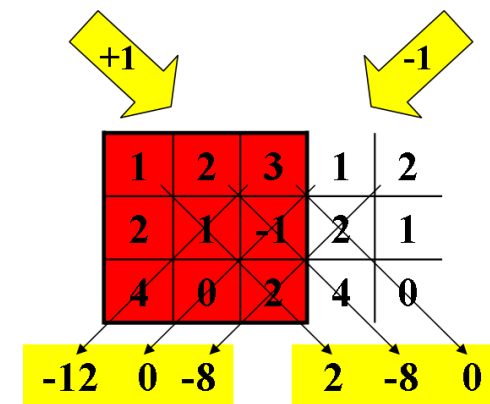
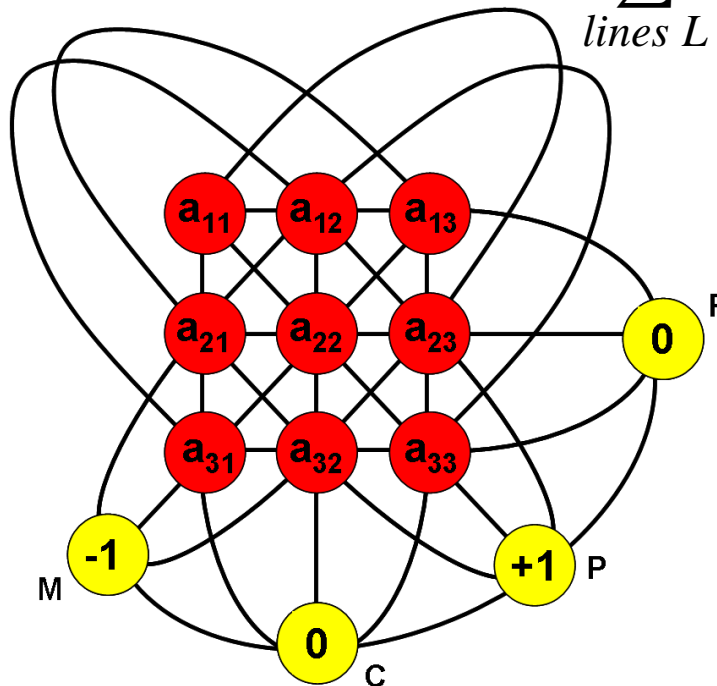
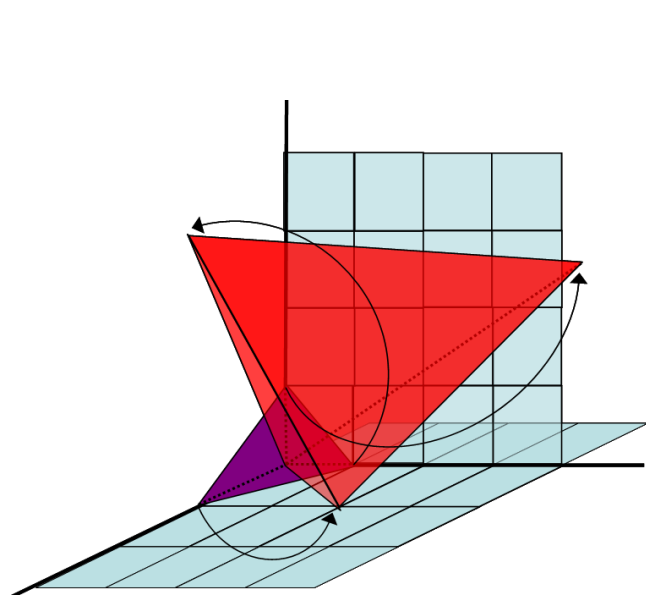




# THEOREM OF THE DAY

**The Rule of Sarrus** In the projective plane of order 3 let the line at infinity consist of points  $R, C, P$  and  $M$ , assigned the values  $0, 0, +1$  and  $-1$ , respectively. Suppose that the other nine points are assigned the entries of a  $3 \times 3$  matrix  $A$  in such a way that the rows of the matrix are assigned to the three parallel lines meeting at  $R$ , the columns to the parallel lines meeting at  $C$ , and the main diagonal to a line through  $P$ . Then the determinant of  $A$  may be evaluated as:  $\det A = \sum_{\text{lines } L} \prod_{x \text{ on } L} (\text{value of point } x)$ .



There are many alternative ways to define the determinant of a matrix  $A$ . Staying within the realm of geometry, we may say that it is the factor (up to sign,  $\pm$ ) by which the volume of a polyhedron is scaled when it is stretched and rotated by  $A$ , viewed as a 'linear transformation'. Above left, a small (purple) tetrahedron of volume  $1/6$  has its points  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$  multiplied by a  $3 \times 3$  matrix, resulting in the large (red) tetrahedron:  $(0, 0, 0)$ ,  $(1, 2, 4)$ ,  $(2, 1, 0)$ ,  $(3, -1, 2)$ . Its volume is  $26 \times 1/6$ , the matrix in question, above right, having determinant  $-26$ , as illustrated by the diagonal multiplications of its elements. (This calculation, in which the first two columns of the matrix are repeated, is the usual way that the Rule of Sarrus is presented; the appeal to projective geometry is gratuitous.)

Pierre Frédéric Sarrus (1798–1861) taught for thirty years at the University of Strasbourg. In 1853 he invented the *Sarrus linkage*, a mechanism for converting rotational motion to linear motion. Eleven years later, a French army officer, Charles-Nicolas Peaucellier, independently solved this problem in mechanics and seems, rather unfairly, to have got all the glory.

**Web link:** [archive.uea.ac.uk/jtm/index.htm](http://archive.uea.ac.uk/jtm/index.htm), Unit 7.2. See [www.theoremoftheday.org/GeometryAndTrigonometry/Sarrus/Sarrus4x4.pdf](http://www.theoremoftheday.org/GeometryAndTrigonometry/Sarrus/Sarrus4x4.pdf) for a  $4 \times 4$  version of the rule, an issue given interesting coverage at [regularize.wordpress.com/2011/06/22/sarrus-rules-for-4-x-4/](http://regularize.wordpress.com/2011/06/22/sarrus-rules-for-4-x-4/).

**Further reading:** *An Elementary Treatise on the Theory of Determinants* by Charles, L. Dodgson, 1867, reprinted Rough House Printing, 2007.

