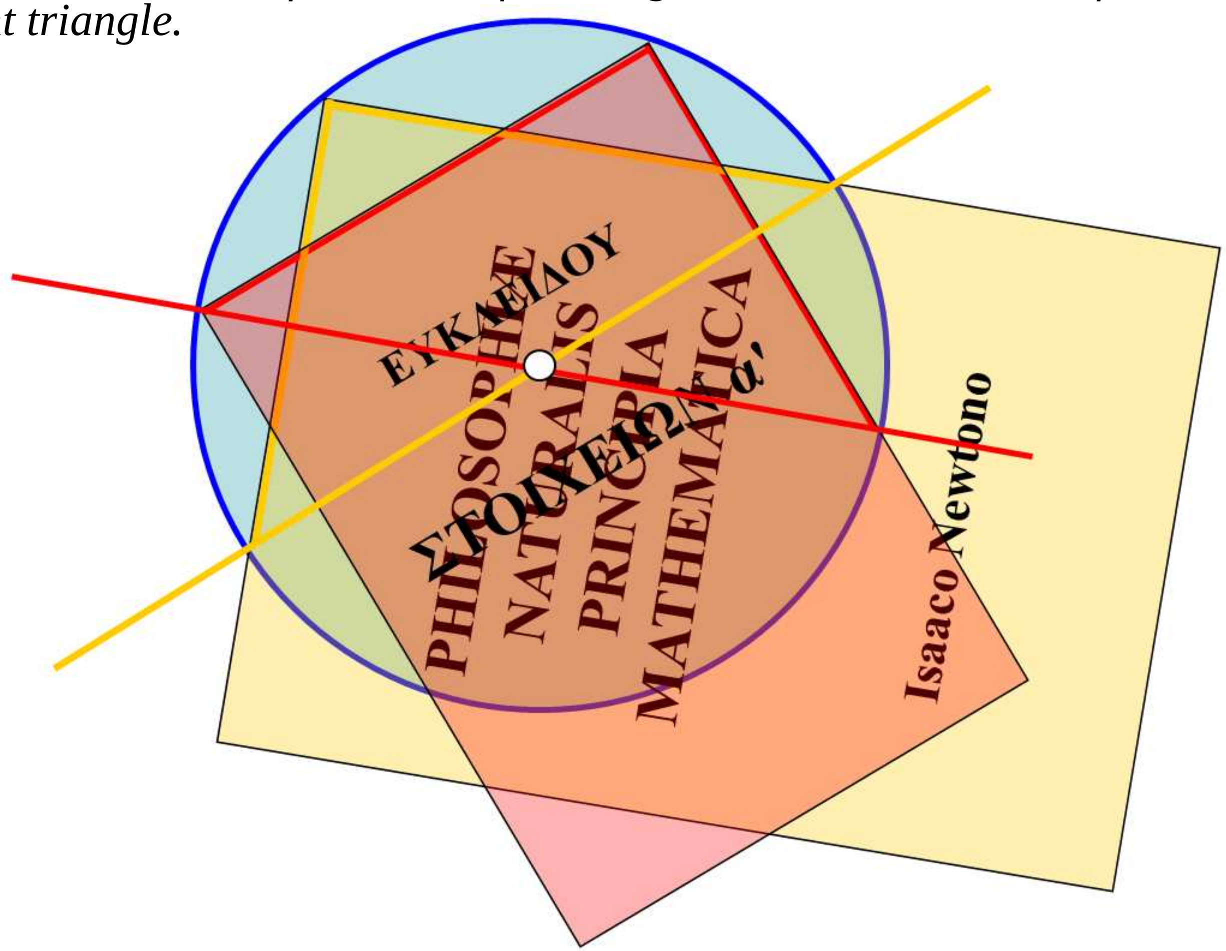
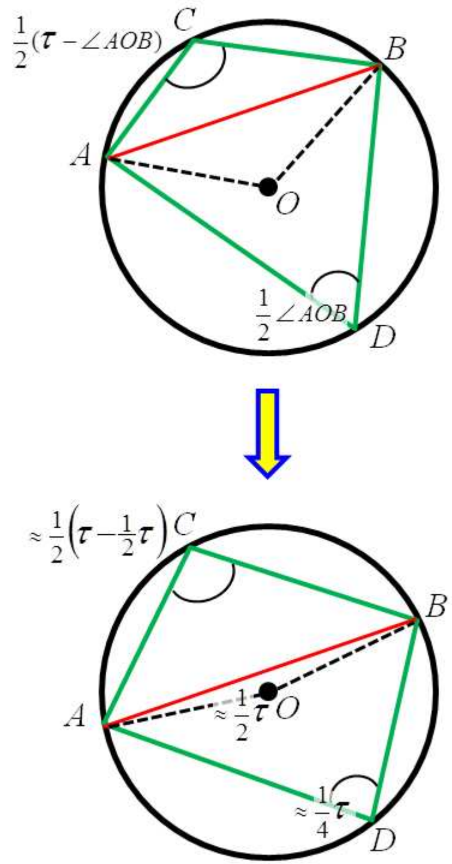
THEOREM OF THE DAY

Thales' Theorem *If one side of a triangle inscribed in a circle forms a diameter of the circle then it is a right triangle.*





Two inscribed right triangles are shown here, illustrating Thales' theorem and its *converse*: that any right triangle inscribed in a circle forms a diameter. This may be used to locate the centre of a circle: place the corner of a book on the circumference and mark where its edges again intersect the circumference. Join these two points with a line. Now rotate the book and repeat to get a second line. The intersection of the two lines is the centre. Thales' theorem generalises to arbitrary triangles inscribed in a circle with centre O as shown above right, using **Euclid, Book III, Proposition 20:** *an angle at O subtending a chord AB is twice any angle on the major arc of the chord and subtending it*. Thus $\angle AOB = 2\angle ADB$. An angle in the *minor* arc is $\tau/2 - \frac{1}{2}\angle AOB$ ($\tau = 2\pi$) because of **Euclid, Book III, Proposition 22:** *opposite angles in a cyclic quadrilateral sum to $\tau/2$*. We recover Thales by letting the length of chord AB approach two radii.

Thales (c.624–547 BC) is said to have sacrificed an ox to mark his discovery of this theorem (a practice lost in modern mathematics). He was the first and perhaps the greatest of the Seven Sages: "Such were Thales of Miletus, and Pittacus of Mitylene, and Bias of Priene, and our own Solon, and Cleobulus the Lindian, and Myson the Chenian; and seventh in the catalogue of wise men was the Spartan Chilon." (Plato, *Protagoras*).

Web link: www.mathopenref.com/thalestheorem.html. The Plato quote is from www.livius.org/articles/people/seven-sages/.
Further reading: *Ancient Mathematics* by Serafina Cuomo, Routledge, 2001.