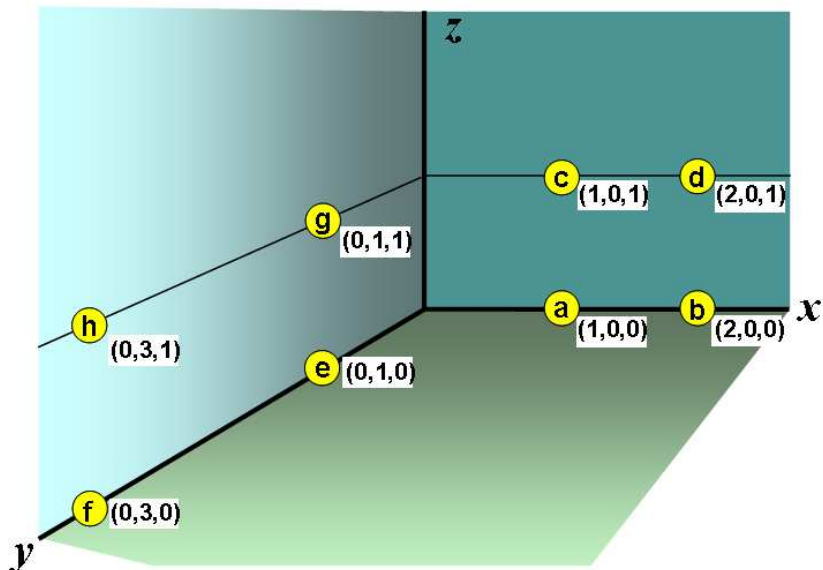




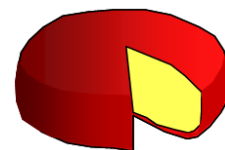
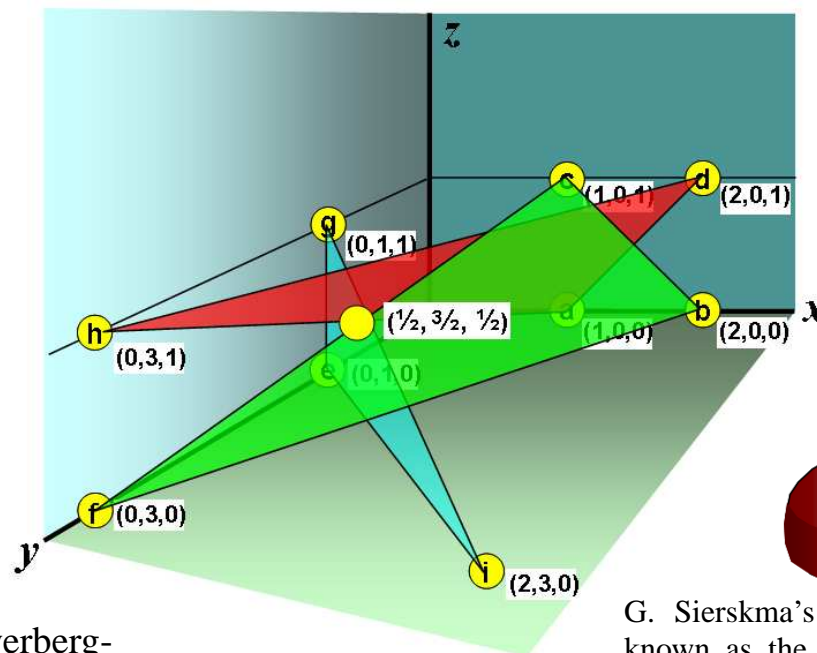
# THEOREM OF THE DAY

**The Tverberg Partition Theorem** A set  $S$  of points in  $d$ -dimensional Euclidean space, whose cardinality exceeds  $(d + 1)(r - 1)$ ,  $r$  a positive integer, can be partitioned into  $r$  subsets,  $S_1, S_2, \dots, S_r$ , whose convex hulls have non-empty intersection:

$$S = \bigcup_{i=1}^r S_i, \quad \text{for } i \neq j, S_i \cap S_j = \emptyset, \quad \bigcap_{i=1}^r \text{conv}(S_i) \neq \emptyset.$$



With  $d = r = 3$ , Tverberg's theorem requires at least  $1 + (3 - 1)(3 + 1) = 9$  points. **Left:** a set of 8 points which cannot be partitioned into three subsets satisfying the conclusion of Tverberg's theorem. **Below:** by adding any single distinct point we can apply Tverberg's theorem. Here the point  $i = (2, 3, 0)$  has been added; the 9 points have been partitioned into 3 subsets of 3 points (whose convex hulls are triangles), intersecting in the point  $(1/2, 3/2, 1/2)$ .



G. Sierksma's conjecture is known as the *Dutch Cheese Problem* because he offered a Dutch cheese for its solution!

There is an important 1979 conjecture of Gerard Sierksma which says that there are at least  $((r-1)!)^d$  partitions satisfying Tverberg's theorem. There are a total of six different 3-triangle partitions of the nine integer points on the right; as an exercise you may like to try to find a further two partitions to reach Sierksma's lower bound of:  $((3 - 1)!)^3 = 8$ .

In 1966 the Norwegian Helge Tverberg published this extension, to arbitrary  $r$ , of the 1921 theorem of Johann Radon (1887–1956) which asserted the case  $r = 2$ . The further study of Tverberg-type partitions has established itself as a flourishing branch of combinatorial geometry.

**Web link:** [www.ams.org/notices/201104/rtx110400550p.pdf](http://www.ams.org/notices/201104/rtx110400550p.pdf) (600KB pdf)

**Further reading:** *Lectures in Discrete Geometry* by Jiří Matoušek, Springer-Verlag, New York, 2002 (chapter 8).

