## THEOREM OF THE DAY

## Mathematical Symbols

Below are brief explanations of some commonly occurring symbols in mathematics presented in more or less haphazard order (the list is not intended to grow so long as to make this irksome).
A word of caution - mathematics has no really fixed rules on what symbols stand for what; never mind that mathematics is supposed to be a highly precise and formal language, reading it relies heavily on contextual information. A $\Sigma$ might be an algebra; $\Sigma_{n}$ might mean the symmetric group; $\Sigma_{i}$ is more likely to mean summation! See http://members.aol.com/jeff570/mathsym.html for a valuable introduction to these issues.

| Symbol | Meaning | Comments |
| :---: | :---: | :---: |
| $\{v \mid C(v)\}$ | set definition | $C(v)$ a membership condition on $v$. E.g. $\{v \mid v$ is an odd number $\}$ |
| $\epsilon$ | set membership | E.g. $2 \in\{x \mid x$ a prime number $\}$ is true; $2 \in\{-1,0,1\}$ is false. |
| $\mathbb{Z}$ | integers | $\{\ldots-3,-2,-1,0,1,2,3, \ldots\}, ~ ' Z$ ' is for the German 'Zahlen' (numbers) |
| $\mathbb{Q}$ | rationals | numbers of form $a / b$ for $a, b \in \mathbb{Z}$ ' Q ' came from German 'Quotient' (ratio) |
| $\mathbb{R}$ | real numbers | the real line: infinitely many digits after the decimal point allowed |
| $i$ | $\sqrt{-1}$ | imaginary number invented to solve $x^{2}+1=0$ |
| $\mathbb{C}$ | complex numbers | numbers of form $a+i b, a, b \in \mathbb{R}, i=\sqrt{-1}$ |
| $\|x\|$ | 'size' | Depending on context: |
|  |  | 1. for $x \in \mathbb{R}$, the value of $x$ ignoring sign; <br> 2. for $x=a+i b \in \mathbb{C}$, the positive value of $\sqrt{a^{2}+b^{2}}$; <br> 3. for $x$ a set, the cardinality of $x$ |

## Symbol

$n$ !
$\binom{n}{k} \quad$ ' $n$ choose $k$ '
$\Sigma \quad$ summation
$e$
$\pi$
$\cong$
$\equiv$
$\aleph_{0}$
$\aleph_{1}$
$\frac{d f}{d x}$
$\int_{a}^{b} f(x) d x \quad$ definite integral of $f$
$\int f(x) d x \quad$ antiderivative of $f$
$\cup$
$\cap$

## Meaning

factorial function
$e=2.7182818285 \ldots$
${ }^{\prime} \mathrm{pi}^{\prime}=3.1415926536 \ldots$
isomorphism
congruence
'aleph nought'
'aleph one'
derivative of $f$
set union
set intersection

## Comments

$n!=1 \times 2 \times \ldots \times(n-1) \times n($ with $0!=1!=1$, by convention $)$ ways to choose $k$ numbers from a set of $n$, given by $\frac{n!}{(n-k)!k!}$
$\sum_{i=1}^{4} x_{i}$ means $x_{1}+x_{2}+x_{3}+x_{4}$
sum of infinite series $\sum_{k=0}^{\infty} 1 / k$ !
ratio of circumference to diameter in a circle
$X \cong Y$ if $X$ and $Y$ have the same size and structure
for integers $x, y$ and $r, x \equiv y(\bmod r)$ if $x$ and $y$ have the same remainder on division by $r$

1st infinite cardinal: the cardinality of $\mathbb{Z}$ (see Glossary)
2nd infinite cardinal: assuming the continuum hypothesis, $\aleph_{1}$ is the cardinality of $\mathbb{R}$ (see Glossary)
the gradiant (instantaneous slope) of function $f(x)$
area under the curve of $f$ from $x=a$ to $x=b$
or indefinite integral: the function of $x$ whose derivative is $f(x)$
$X \cup Y$ is the set containing everything found in either $X$ or $Y$
$X \cap Y$ is the set containing everything found in both $X$ and $Y$

| Symbol | Meaning | Comments |
| :--- | :--- | :--- |
| $x \rightarrow y$ | ' x tends to y ' | the distance between $x$ and $y$ is allowed to grow arbitrarily small. <br> E.g. $n \rightarrow \infty$. |
| $f: X \rightarrow Y$ | function | function $f$ maps set $X$ to set $Y$ |
| $\chi(G)$ | chromatic number | minimum number of colours allowing graph $G$ to be properly <br> coloured (see Glossary) |

