



# THEOREM OF THE DAY

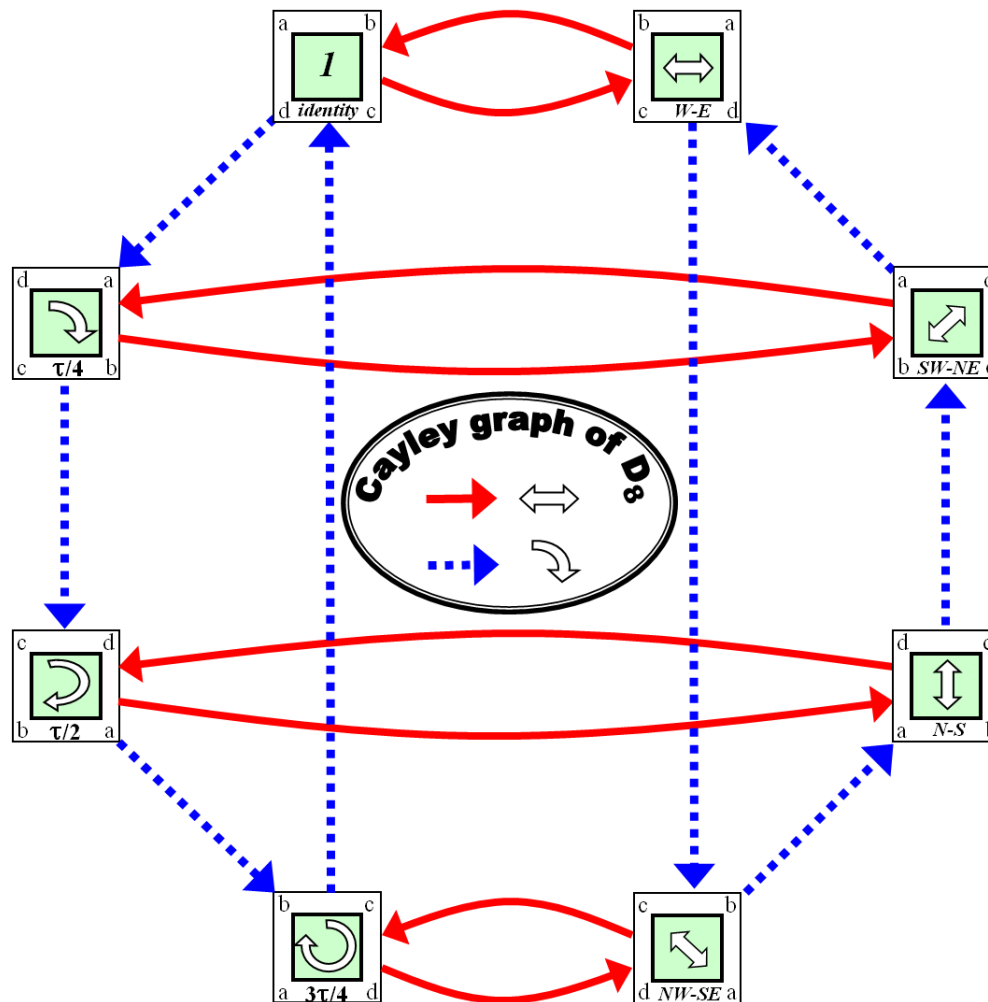
**Cayley's Theorem** A finite group of order  $n$  is isomorphic to a subgroup of the symmetric group on  $n$  points.

$\times$	1							
identity	1							
rotate $\tau/4$				1				
rotate $\tau/2$			1					
rotate $3\tau/4$		1						
NW-SE flip				1				
N-S flip						1		
SW-NE flip							1	
W-E flip								1

$\tau = 1$  turn ( $=2\pi$ )

**A  $D_8$  times-table**

The symmetric group  $S_8$  consists of all permutations of 8 objects. Each row and column of the multiplication table for  $D_8$ , the symmetries of the square, clearly constitutes a permutation of the 8 possible symmetries. In fact, we can identify two symmetries which, by repeated multiplication, generate all others. This may be viewed pictorially as a so-called *Cayley graph*, as shown on the right: rotation by a quarter-turn permutes  $D_8$  in two 4-cycles; the West–East flip in four 2-cycles.



Although Cayley's Theorem is classic textbook stuff, I have recorded no less an authority than Peter M. Neumann referring to it as "an important theorem in the nineteenth century; a pretty feeble theorem in the twentieth; an even more feeble theorem now." Arthur Cayley published the theorem in 1854 in one of the first ever papers on group theory. At that time, Neumann said, it would have been a kind of 'comfort blanket' for mathematicians unfamiliar with the abstract idea of a group.

**Web link:** [www-history.mcs.st-and.ac.uk/~edmund/lnotes/node14.html](http://www-history.mcs.st-and.ac.uk/~edmund/lnotes/node14.html)

**Further reading:** *Groups and Geometry* by Peter M. Neumann, Gabrielle A. Stoy and Edward C. Thomson, OUP, 1994.

