The Cameron–Fon-Der-Flaass IBIS Theorem Let $G$ be a permutation group acting on a set $\Omega$. Then the following are equivalent:

1. all irredundant bases of $G$ have the same size;
2. the irredundant bases of $G$ are preserved by re-ordering;
3. the irredundant bases of $G$ form the bases of a matroid.

$\Omega = \{a, b, c, d, e, f\}$

$G = \langle g_1 = (a, f), \quad g_2 = (b, d)(c, e), \quad g_3 = (b, c, e, d) \rangle$

$H = \langle h_1 = (a, f)(b, d)(c, e), \quad h_2 = (b, c), \quad h_3 = (d, e) \rangle$

The group $G$ represents the 16 symmetries of the irregular octahedron shown on the left. A base is a sequence of vertices which, when fixed, destroys all symmetry. Fixing vertex $a$ removes $g_1$, the vertical rotation; fixing $b$ removes $g_2$ and $g_3$ but not $g_2g_3 = (c, d)$; fixing $c$ removes this last symmetry; and $[a, b, c]$ is a base. Or we could finish by fixing $d$: so another base is $[a, b, d]$. In fact, in the graph $M$ above-centre, any three edges which meet all three vertices, taken in any order, correspond to a base. Each of these bases is irredundant: each element of each sequence plays its part in removing additional symmetry. Moreover, $M$ gives us a matroid: take any two bases $B_1$ and $B_2$ and they obey the ‘exchange property’ that we can replace some element in $B_1 \setminus B_2$ with an element in $B_2 \setminus B_1$ and again get a base. Not all groups have such well-behaved bases! The 8 symmetries of the figure on the right form the group $H$. Now $[a, b, d]$ is an irredundant base but the reordering $[b, a, d]$ is not irredundant: with $b$ fixed, no remaining symmetries move $a$, so $[b, d]$ is a smaller base.

Peter Cameron and Dima Fon-Der-Flaass made this too-good-to-be-true link between group theory and combinatorics in 1995. A group satisfying its three equivalent properties is called an IBIS group: it has ‘Irredundant Bases of Invariant Size’.

Web link: cameroncounts.github.io/web/talks/; see “Beyond matroids?” 2015.