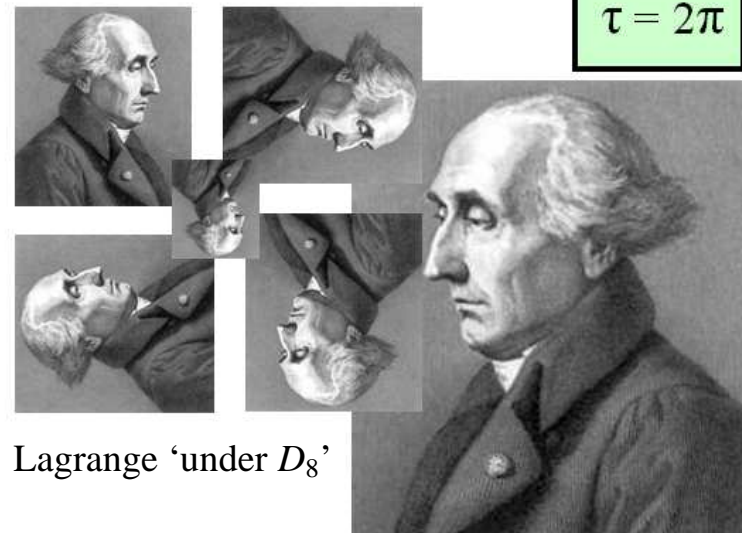
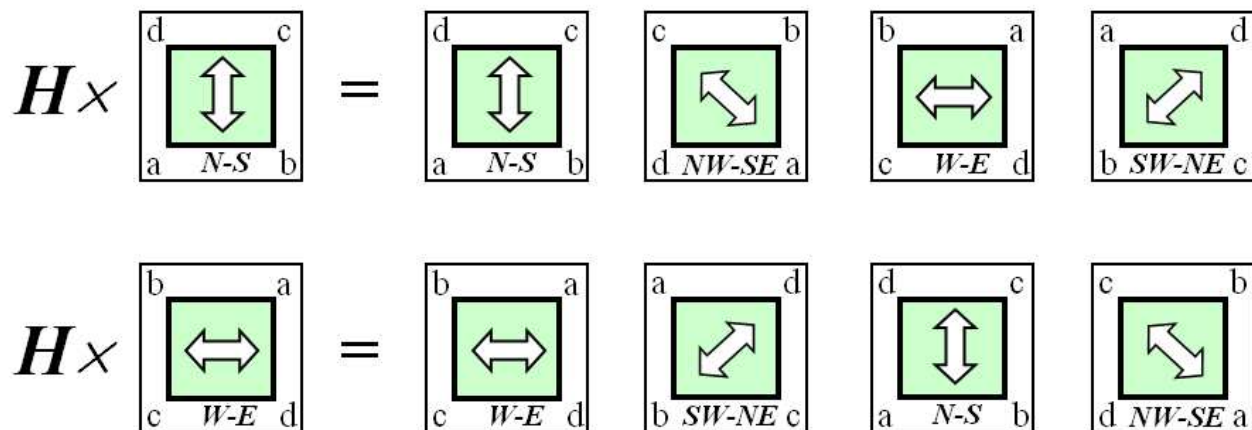
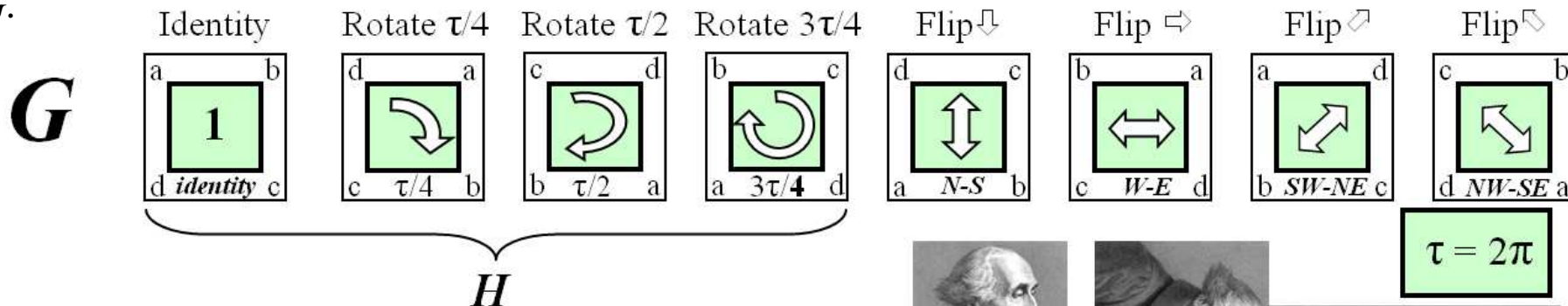




# THEOREM OF THE DAY

**Lagrange's Theorem** If  $G$  is a finite group and  $H$  is a subgroup of  $G$  then the order of  $H$  divides the order of  $G$ .



In the picture,  $G$  is the *dihedral group*  $D_8$ : the group of symmetries of the regular 4-sided polygon (the square). It has order 8.  $H$  is the subgroup of order 4 of rotational symmetries. When we multiply each element of  $H$  by the horizontal mirror symmetry (identity then flip, rotate  $\tau/4$  then flip, etc) we get a *right coset* of  $H$  in  $G$ , no longer a subgroup (since the identity has gone) but still with the same number of distinct elements. Now multiply  $H$  by vertical mirror symmetry: we get (a rearrangement of) exactly the same coset. Try multiplying  $H$  by the diagonal mirror symmetries: you get rearrangements of this same coset. This illustrates two facts which, taken together, establish the theorem: all cosets of  $H$  (a) have  $|H|$  elements; and (b) are identical or disjoint.

In 1771, the Italian-French mathematician Joseph-Louis Lagrange (1736–1813) published, without proof, a special case of this theorem relating to permutations of polynomial roots. It took 100 years to arrive at its general form, as given above, which may be attributed to Camille Jordan, although Cauchy gave a proof for subgroups of the symmetric group in 1812.

**Web link:** [www-history.mcs.st-and.ac.uk/~edmund/lnotes/node12.html](http://www-history.mcs.st-and.ac.uk/~edmund/lnotes/node12.html), and Janet Barnett will guide you through a wonderful historical investigation here: [www.cs.nmsu.edu/historical-projects/projects.php](http://www.cs.nmsu.edu/historical-projects/projects.php), see Project 11.

**Further reading:** *Groups and Geometry* by Peter M. Neumann, Gabrielle A. Stoy and Edward C. Thomson, Oxford University Press, 1994.

