THEOREM OF THE DAY

The Robbins Problem Every Robbins algebra is a Boolean algebra.

<table>
<thead>
<tr>
<th>BOOLEAN ALGEBRA</th>
<th>ROBBINS ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutation</td>
<td>$a + b = b + a$</td>
</tr>
<tr>
<td>Association</td>
<td>$(a + b) + c = a + (b + c)$</td>
</tr>
<tr>
<td>Zero</td>
<td>$0 + a = a + 0 = a$</td>
</tr>
<tr>
<td>One</td>
<td>$1 + a = a + 1 = 1$</td>
</tr>
<tr>
<td>Distribution</td>
<td>$a + (b + c) = (a + b) + (a + c)$</td>
</tr>
<tr>
<td>Absorption</td>
<td>$a \times (a + b) = a + (a \times b) = a$</td>
</tr>
<tr>
<td>Complementation</td>
<td>For every $a$ there is some $a'$ satisfying $a \times a' = 0$ and $a + a' = 1$</td>
</tr>
</tbody>
</table>

News Flash!

McCune, Veroff, Fitelson, Harris, Feist and Wos publish short single-axiom specification of Boolean Algebra, J. Automated Reasoning, 29(1), 1–16, 2002: $(((x+y)+z)' + (x+(z'+(z+u)'))) = z$

A Boolean algebra is a set containing at least two elements, 0 (zero) and 1 (one); closed under three operations, $+, \times$ and $'$ (complement); and satisfying the laws given in the table, above left. These are exactly the laws of ordinary (integer) arithmetic, until we reach the four that are shaded (red), which take us into a different realm — that of logic and set theory. Amazingly, this whole realm may then be collapsed down to just three laws, specifying the Robbins algebra, above top right. Now, while it is simple to see that any Boolean algebra is a Robbins algebra, the reverse derivation is extraordinarily difficult. The inset, bottom right, is part of a summary of the proof, by the theorem prover EQP, that every Robbins algebra satisfies the so-called 2nd Winker condition: $\exists C, D.(C + D)' = C'$, already known to guarantee the laws of Boolean algebra.

Herbert Ellis Robbins (1915–2001), better known as a statistician, influenced logic, mathematics and computer science by proposing a slight simplification of an algebra due to Edward Vermilye Huntington in 1933. Proving that this was equivalent to Boolean algebra, the Robbins Problem, stone-walled human logicians for decades. A breakthrough came in 1979, when Larry Wos and Steve Winker at the Argonne National Laboratory, using the fledgling methods of automated reasoning, discovered the Winker conditions. Even then, it took nearly twenty more years before William McCune, in collaboration with his EQP theorem prover, finally completed the solution in October 1996. This was a major milestone in artificial intelligence.

Web link: www.cs.unm.edu/~mccune/papers/robbins/ (links to the EQP output which is reproduced in part above).