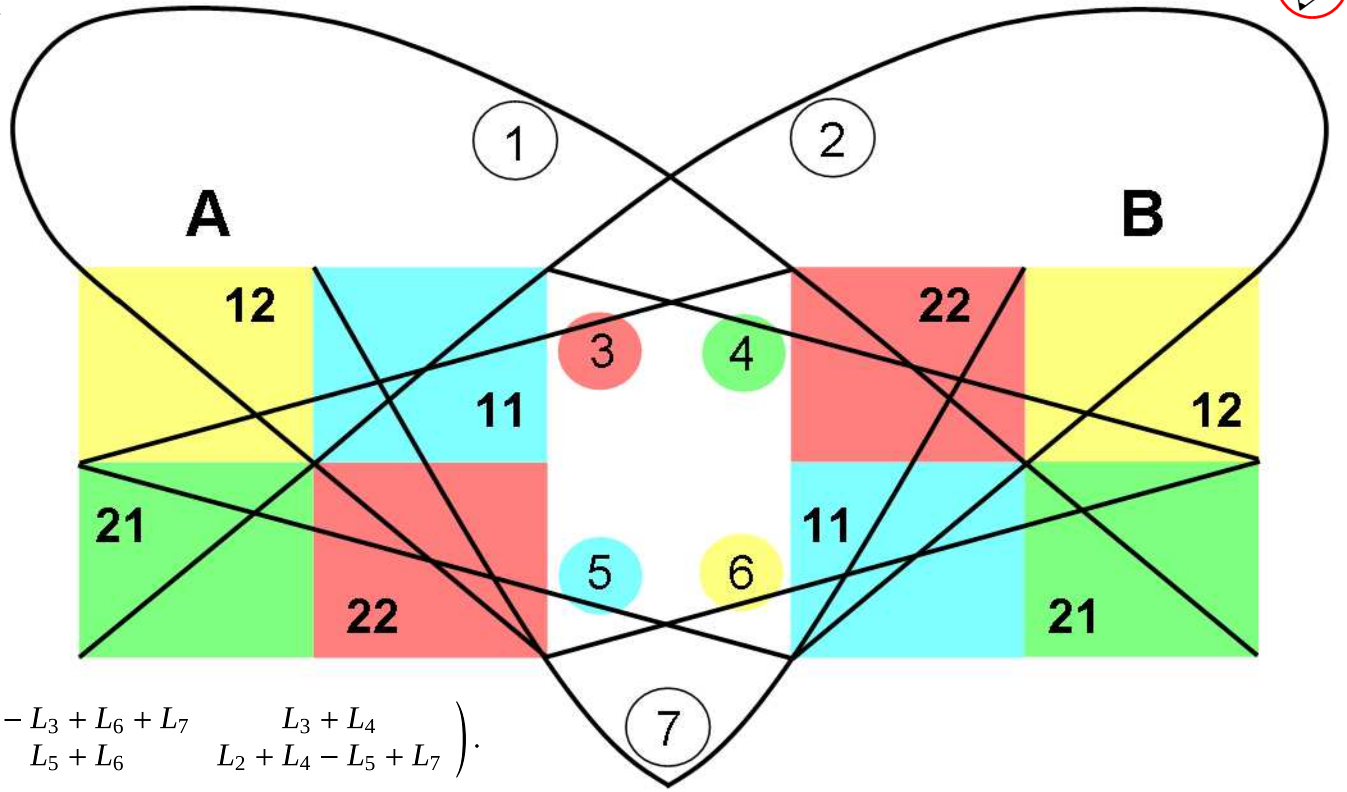
THEOREM OF THE DAY

Strassen's Matrix Theorem (a Theorem Under Construction!) *Two $n \times n$ matrices can be multiplied in fewer than n^3 (multiplication) steps.*



Instructions for using this diagram

1. Entries of matrices A and B are arranged as shown. Now seven products are formed as follows:
2. Entries along curved lines (L_1, L_2 and L_7, passing four entries) and straight lines (L_3, L_4, L_5 and L_6, passing three) are added *within each matrix* and the sums are multiplied *between the matrices*. Line L_7 gives $(a_{11} + a_{22}) \times (b_{11} + b_{22})$, for example. *But ...*
3. ... where green (21) and blue (11) appear consecutively, the blue entry is subtracted rather than added; thus L_6 gives $a_{22}(b_{21} - b_{11})$; *and ...*
4. ... a red (22) consecutively with yellow (12) is likewise subtracted: L_1 gives $(a_{12} - a_{22})(b_{21} + b_{22})$.
5. Now $AB = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix} = \begin{pmatrix} L_1 - L_3 + L_6 + L_7 & L_3 + L_4 \\ L_5 + L_6 & L_2 + L_4 - L_5 + L_7 \end{pmatrix}$.
6. For larger matrices (padded out to size $2^k \times 2^k$ as necessary) partition into square blocks and proceed as above.

Construction notes:

1969: Volker Strassen amazes the world by multiplying $n \times n$ matrices in $O(n^{2.808...})$ (multiplication) steps.

1970s & 80s: Strassen's result leads to a succession of lower values for the exponent, which becomes known as ω.

1987: Don Coppersmith and Shmuel Winograd achieve best value for ω to date with $O(n^{2.376...})$ algorithm.

2003: Chris Umans and Henry Cohn discover group theoretic formulation of Strassen's approach.

2005: With Robert Kleinberg and Balazs Szegedy they reproduce Coppersmith and Winograd group theoretically and give two conjectures whose proof will reduce ω to its theoretical minimum value of 2.

2011: Virginia Vassilevska Williams reduces ω to 2.373. Meanwhile, Strassen's original method remains the fastest practical way to multiply matrices larger than around 30×30.

Web link: an excellent overview: archive.siam.org/news/news.php?id=174; click the icon (top right) for news of the latest developments.

Further reading: *Algorithms and Complexity, 2nd edition* by Herbert S. Wilf, A K Peters, 2003.