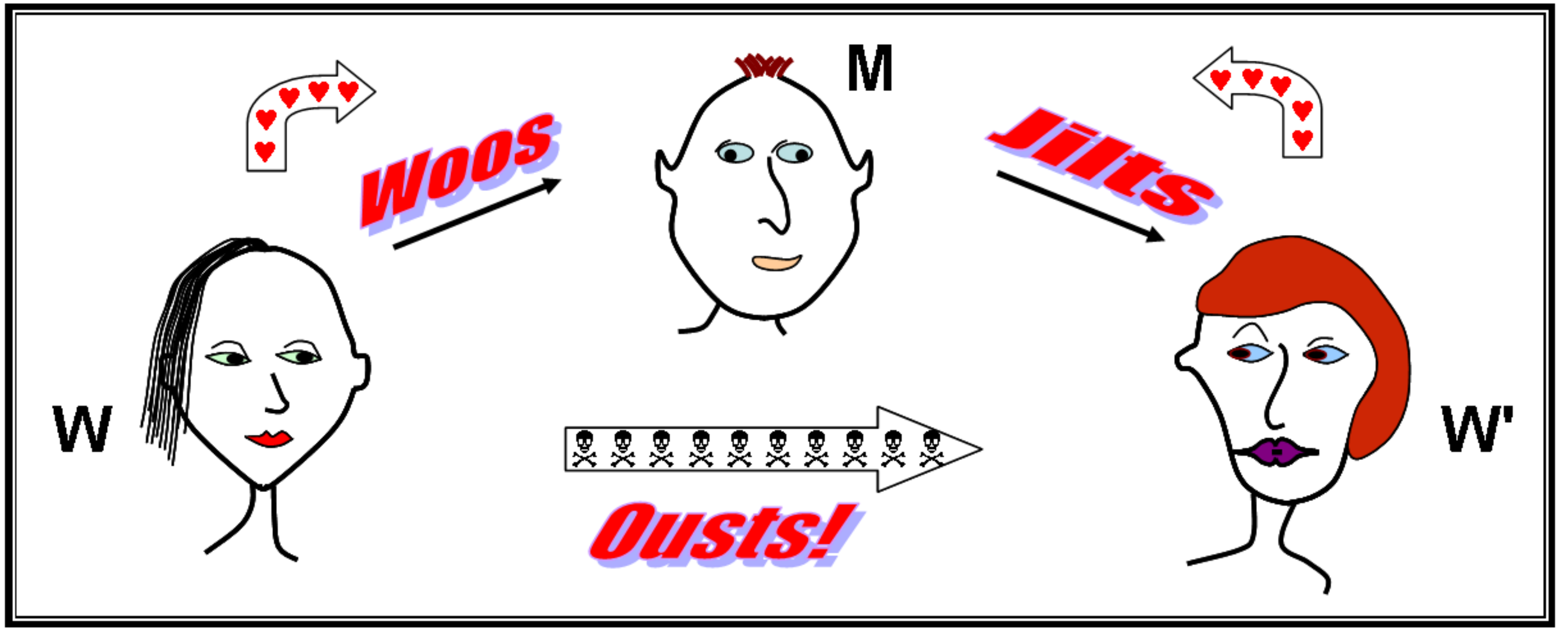
THEOREM OF THE DAY

The Stable Marriage Theorem *Suppose n women rank n men in order of preference. The men, likewise, rank the n women. Then there exists a* stable marriage*: a pairing of the women and men such that no pair exists who would rather be married to each other than to their assigned partners.*



The old love-triangle! A stable marriage is found by repeatedly applying the following steps: let some as-yet unattached woman W choose her highest ranked male, M, say. If M is unattached, she gets him. If he is already engaged to W' then, if he ranks W higher, he jilts W' for W and W' must cross him off her list; otherwise he sticks with W' and it is W who crosses him off *her* list.

Example:

	M_1	M_2	M_3	M_4
W_1	1 \ 3	2 \ 1	2 \ 3	3 \ 1
W_2	2 \ 1	1 \ 4	3 \ 2	4 \ 4
W_3	1 \ 4	2 \ 2	4 \ 1	3 \ 2
W_4	1 \ 2	1 \ 3	3 \ 4	2 \ 3

In this table four women have ranked four men (the numbers in the bottom-left triangles) and the men have ranked the women (the top-right triangles). Note that ties are allowed. An unstable marriage would be (W_1, M_1), (W_2, M_2), (W_3, M_3), (W_4, M_4), since M_2 and W_3 clearly hate their partners and will run off with each other! If you iterate the above process starting with woman W_1, you should find W_1, W_2 and W_3 are all jilted before finding a stable partnership. The stable marriage is: (W_1, M_2), (W_2, M_1), (W_3, M_4), (W_4, M_3). Poor W_4 and M_3 mutually dislike each other but everyone else is stably married so they must remain contented.

This 1961 theorem and algorithm of two famous American mathematical economists David Gale and Lloyd Shapley, has a potential application wherever an assignment or allocation of people or organisations must be made.

Web link: www.ams.org/samplings/feature-column/fcarc-marriage

Further reading: *The Stable Marriage Problem: Structure and Algorithms* by D. Gusfield and R.W. Irving, MIT Press, 1989.