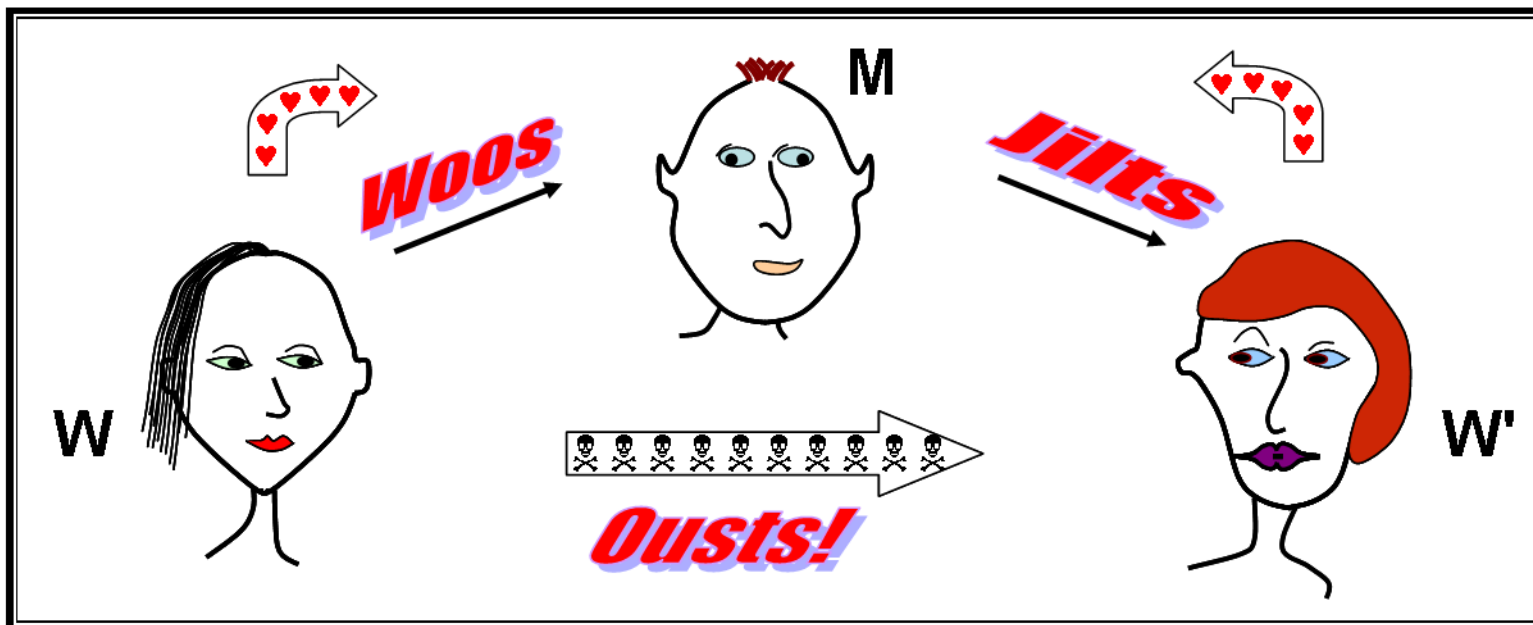




# THEOREM OF THE DAY

**The Stable Marriage Theorem** Suppose  $n$  women rank  $n$  men in order of preference. The men, likewise, rank the  $n$  women. Then there exists a stable marriage: a pairing of the women and men such that no pair exists who would rather be married to each other than to their assigned partners.



The old love-triangle! A stable marriage is found by repeatedly applying the following steps: let some as-yet unattached woman  $W$  choose her highest ranked male,  $M$ , say. If  $M$  is unattached, she gets him. If he is already engaged to  $W'$  then, if he ranks  $W$  higher, he jilts  $W'$  for  $W$  and  $W'$  must cross him off her list; otherwise he sticks with  $W'$  and it is  $W$  who crosses him off *her* list.

**Example:**

	$M_1$	$M_2$	$M_3$	$M_4$
$W_1$	1	3	2	1
$W_2$	2	1	1	4
$W_3$	1	4	2	2
$W_4$	1	2	1	3

In this table four women have ranked four men (the numbers in the bottom-left triangles) and the men have ranked the women (the top-right triangles). Note that ties are allowed. An unstable marriage would be  $(W_1, M_1)$ ,  $(W_2, M_2)$ ,  $(W_3, M_3)$ ,  $(W_4, M_4)$ , since  $M_2$  and  $W_3$  clearly hate their partners and will run off with each other! If you iterate the above process starting with woman  $W_1$ , you should find  $W_1$ ,  $W_2$  and  $W_3$  are all jilted before finding a stable partnership. The stable marriage is:  $(W_1, M_2)$ ,  $(W_2, M_1)$ ,  $(W_3, M_4)$ ,  $(W_4, M_3)$ . Poor  $W_4$  and  $M_3$  mutually dislike each other but everyone else is stably married so they must remain contented.

This 1962 theorem and algorithm of two famous American mathematical economists David Gale (1921-2008) and Lloyd Shapley, has a potential application wherever an assignment or allocation of people or organisations must be made.

**Web link:** [www.ams.org/samplings/feature-column/fcarc-marriage](http://www.ams.org/samplings/feature-column/fcarc-marriage)

**Further reading:** *The Stable Marriage Problem: Structure and Algorithms* by D. Gusfield and R.W. Irving, MIT Press, 1989.

