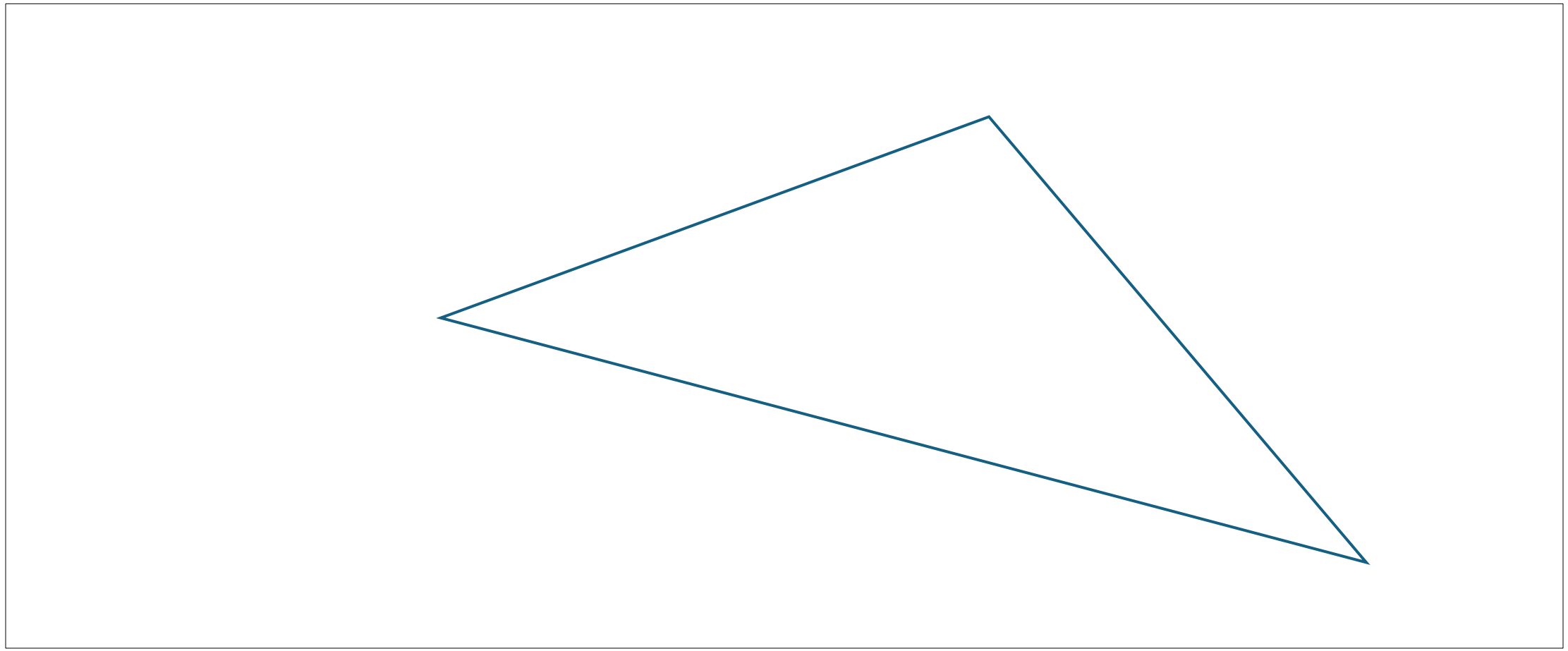
Do Polygons Count?

Source: Branko Grünbaum, *Polygons* pp 147 – 164 (Lecture Notes in Mathematics 490, *The Geometry of Metric and Linear Spaces,* L.M. Kelly (Ed.), Springer, 1975}

But First There Is One:



These are representative FORMS of POLYGONS

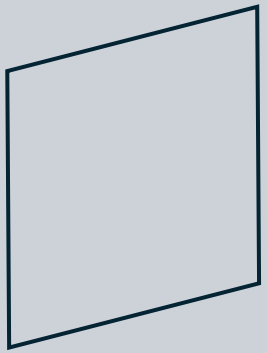
- For *n*=3 there is only 1 form

  TRIANGLE

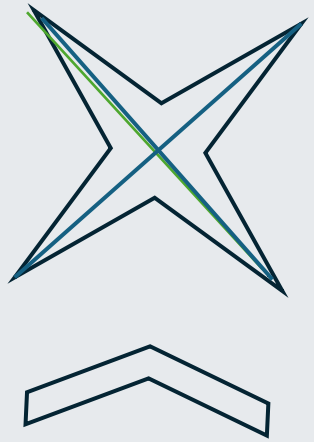
- For n=4 there are only 3 forms
  - BOX, BOOMERANG, BOW TIE

For *n=4* there are only three::

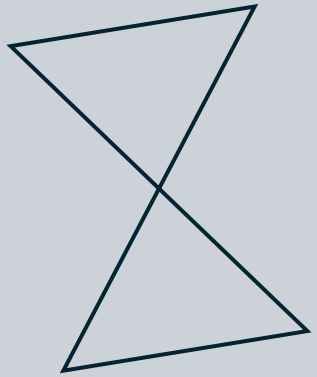
BOX



BOOMERANG



BOW TIE



FORMS of POLYGONS

- For n=3 there is only 1 form.
- For n=4 there are only 3 forms
- For n=5 there are only 11 forms
- For n=6 there are only 72 forms
- For n=7 there are only ? forms

WHAT IS A POLYGON FORM?

- BUT FIRST
- WHAT IS A POLYGON?

A POLYGON IS A:

- GRAPH
- CONNECTED
- 2-REGULAR
- PLANAR

A POLYGON IS A GRAPH

- CONNECTED, 2-REGULAR, PLANAR, AND
- EDGES ARE STRAIGHT
- VERTICES ARE NOT STRAIGHT

A POLYGON IS A GRAPH

- CONNECTED, 2-REGULAR, PLANAR, AND
- EDGES ARE STRAIGHT
- VERTICES ARE NOT STRAIGHT
- SELF-INTERSECTION RESTRCTED

A POLYGON IS A GRAPH

- CONNECTED, 2-REGULAR, PLANAR, EDGES ARE STRAIGHT, VERTICES ARE NOT STRAIGHT, SELF-INTERSECTION RESTRCTED.
- NO POINT LIES ON MORE THAN TWO EDGES.

  THUS: No three edges have a common point
  - No two edges share a segment of positive length.
  - No vertex lies on the interior of an edge.
  - No two vertices coincide.

A POLYGON IS A GRAPH

- CONNECTED, 2-REGULAR, PLANAR, EDGES ARE STRAIGHT, VERTICES ARE NOT STRAIGHT, SELF-INTERSECTION RESTRCTED.
- NO POINT LIES ON MORE THAN TWO EDGES.
- ASSUME:
- The number of vertices, n, $3 \leqq n < \infty$.

WHAT IS A POLYGON FORM?

- IT IS AN EQUIVALENCE RELATION ON POLYGONS
- Two n-gons, P = $[x_1,\ldots,x_n]$ and P' = $[x_1',\ldots,x_n']$ have the same form if there is a family of n-gons P(t) = $[x_1(t),\ldots,x_n(t)]$ for $0\le t\le 1$ such that P(0)=P and P(1)=P' or the mirror image of P' and for each $j=1,2,\ldots,n$ the point $x_j(t)$ is continuous function of t in the interval $0\le t\le 1$.
- Each of the n-gons P(t) must satisfy all the conditions of being a polygon.

$n = 6$

# Self Inter-sections	Number of Concavities			
	0	1	2	3
0	1	1	3	3
1	2	4	11	3
2	3	5	2	1
3	4	5	6	2
4	0	3	3	0
5	1	4	1	0
6	1	1	1	0
7	1	0	0	0