Writing Numbers in Base N (Number Systems and Just Touching Covering Systems)

Gary Michalek (La Salle)

August 31, 2016

Nonnegative integers can be written in base three using the numbers 0, 1 and 2 as coefficients (The set $A = \{0, 1, 2\}$ contains each possible remainder when dividing by 3). For example 102 stands for $1\left(3^2\right) + 0\left(3^1\right) + 2\left(3^0\right)$ which is the way to write the number 11.

If you form $B = A - A = \{0, 1, -1, 2, -2\}$, then all integers (negative, positive or 0) can be written as a sum of powers of three with coefficients in B. Do other sets of numbers work like this? For example, the remainders when dividing by 3 can be represented equally well by the set $A = \{0, 1, 5\}$. Does $B = A - A$ still suffice to express every integer? What can we say about bases other than three? We will present a theorem from 2001 that answers this, and in general shows what numbers are expressible using B and which are not. The result is quite easy. The majority of the talk will be an explanation of the proof, which has some perhaps unanticipated consequences for the way a number can be expressed. While the proof is somewhat involved, the mathematical background required is minimal. The most technical parts of the proof involve subgroups and quotient groups of the integers $\mathbb{Z}$, both of which will be simply explained.

A more complicated and less illuminating proof was published by the author in "Base N Just Touching Covering Systems" (*Publicationes Mathematicae Debrecen*, Tomus 58/4 (2001), pp. 549–557).

London South Bank University, Maths Study Group, 7 September 2016