Matrices, exterior algebra and combinatorics

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A square matrix M can be regarded either as a combinatorial object or as the representation of a linear map on some vector space, say V. The (vague) question arises: what can we say about the geometric or linear algebraic properties of M, such as its spectrum, from its combinatorial structure encoded in some graph, say G? A number of elegant results in this direction are known where natural computations on graphs replace awkward ones on matrices. However, one area which seems little explored involves taking an indirect approach and considering actions of M beyond the obvious action on V. Here is where exterior algebra comes in: for example certain computations on G can provide information about an action of M on some exterior power of V, which in turn tells us something new about M. I'll describe the basics of exterior algebra and give examples of how these approaches can give useful — even surprising — results.