**The Friedlander–Iwaniec Theorem**

There are infinitely many prime numbers of the form $m^2 + n^4$, for positive integers $m$ and $n$.

Number theorists are morally certain that any reasonable polynomial $f(x_1, \ldots, x_t)$, in several positive integer variables and with integer coefficients, will take infinitely many prime values. Of course $f$ must not factorise over the rationals, and there are obvious so-called ‘local conditions’, e.g. $f(x) = x(x + 1) + 2$ is excluded because one of $x$ and $x + 1$ is even, forcing $f(x)$ to be even. To start with the one-variable linear prototype: $f(x) = ax + b$ produces infinitely many primes if and only if $a$ and $b$ are coprime. Legendre asserted this in 1785; its proof sixty years later by Dirichlet marks the birth of analytic number theory.

Friedlander and Iwaniec used sophisticated prime ‘sieving’ methods to give the first proof that a thin polynomial sequence could contain infinitely many primes, inspiring Heath-Brown’s proof that there are infinitely many primes which are sums of three cubes.

**Web link:** Iwaniec and Friedlander: [www.pnas.org/content/94/4/1054.abstract](http://www.pnas.org/content/94/4/1054.abstract); Heath-Brown: [projecteuclid.org/euclid.acta/1485891369](http://projecteuclid.org/euclid.acta/1485891369).


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