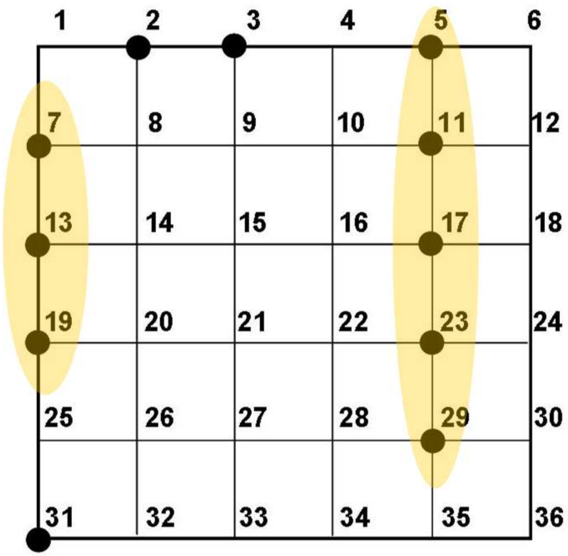
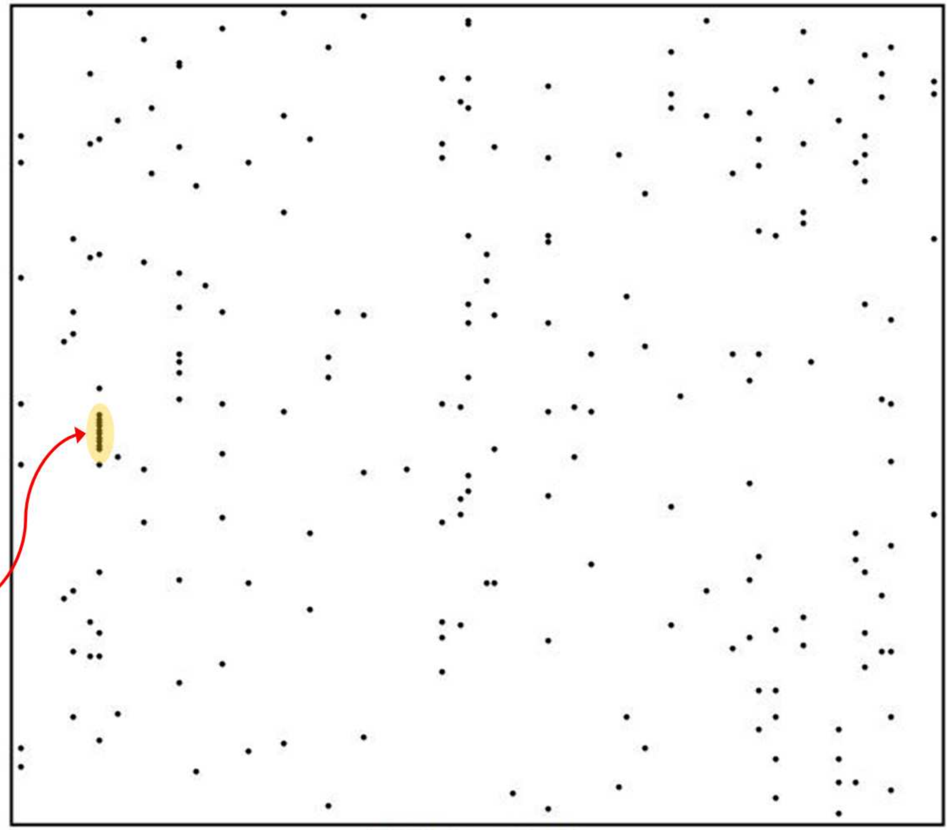
THEOREM OF THE DAY

The Green–Tao Theorem on Primes in Arithmetic Progression *For any positive integer k there exist infinitely many arithmetic progressions of length k consisting of prime numbers.*



10 consecutive primes in arithmetic progression

The image is due to Tony Forbes, a member of the team who found the sequence in 1998



On the left we use a 6×6 grid to locate a sequence of primes equally spaced at distance 6. Thus 7, 13, 19 forms an *arithmetic progression* with *common difference* 6. A longer arithmetic progression is 5, 11, 17, 23, 29. The 210×210 grid on the right shows the record-breaking sequence of 10 *consecutive* primes in arithmetic progression, discovered in March 1998 by Manfred Toplic, who was part of a one hundred-strong team, using over 200 computers. Instead of starting at 1 in the top-left hand corner, this grid starts with a 93 digit number, since the first prime in the sequence is 100 99697 24697 14247 63778 66555 87969 84032 95093 24689 19004 18036 03417 75890 43417 03348 88215 90672 29719.

Note that Green and Tao's theorem, announced in 2004, does not assert that arbitrarily long progressions of *consecutive primes* must exist. This would be a huge strengthening of their result, already a stunning achievement.

Web links: terrytao.wordpress.com/2009/08: click on the 3rd of Tao's wonderful 2009 Mahler Lectures. Tony Forbes' image is from anthony.d.forbes.googlepages.com/10primes.htm (page opens as an attachment).

Further reading: *Elementary Number Theory* by Gareth Jones and Mary Jones, Springer, Berlin, 1998.