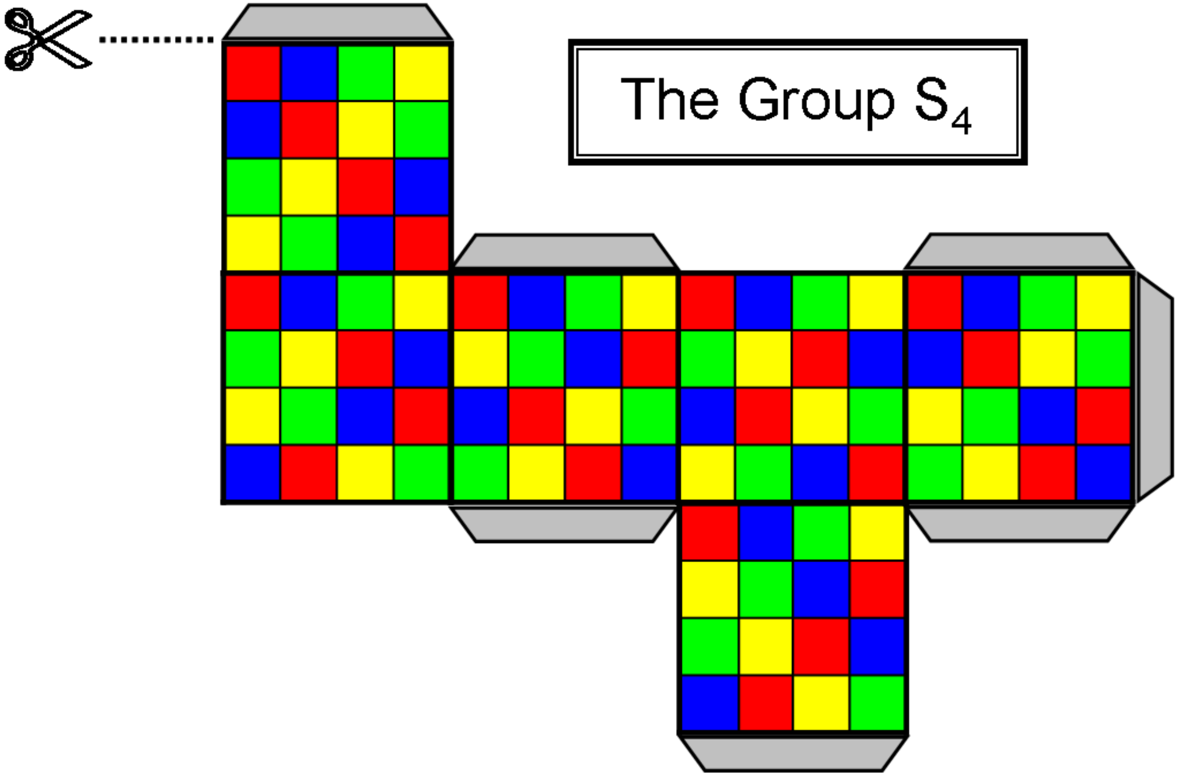
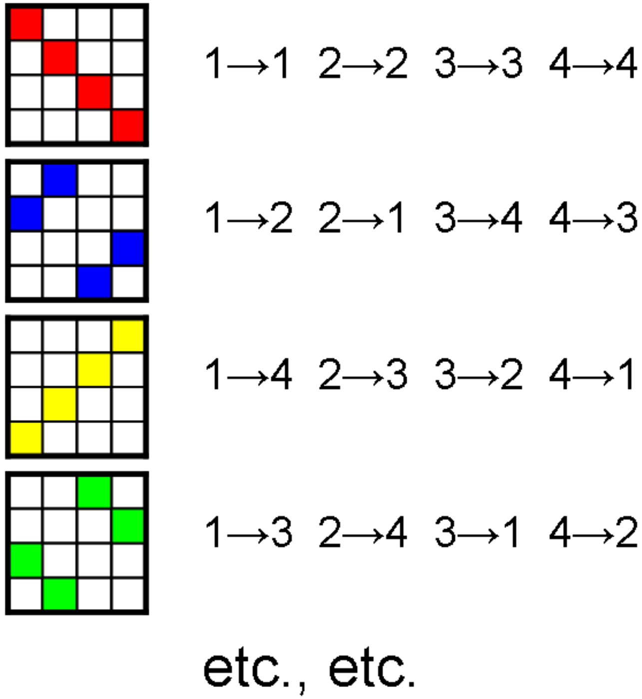
An Erdős-Ko-Rado Theorem on Intersecting Permutations *Let $\mathcal{A}$ be a family of permutations of the set* $[n] = \{1, \ldots, n\}$, $n \geq 2$. *Suppose $\mathcal{A}$ is* intersecting, *that is, for any pair of permutations f and g in $\mathcal{A}$ there is a point $x \in [n]$ such that $f(x) = g(x)$. Then $|\mathcal{A}| \leq (n-1)!$ with equality precisely when $\mathcal{A}$ is a coset of some point stabilizer (for some $x, y \in [n]$, $\mathcal{A} = \{f \mid f(x) = y\}$).*



The Group S_4



Cut out and fold to make a pattern for solving this Rubik's Revenge cube with a difference!

Each colour on each face forms a permutation. Moreover, each colour forms a maximal intersecting family, of size (4-1)!=6. As displayed above, all the reds share the same top-left position and form the stabilizer of 1: $\{f \mid f(1) = 1\}$, the subgroup of all permutations fixing 1. Meanwhile, the other colours form cosets of the stabilizer, fixing the other positions in the top row.

The upper bound in this theorem was proved by Michel Marie Deza and Péter Frankl in 1977, making an analogy with a famous 1938 theorem on intersecting families of sets due to Erdős, Ko and Rado. Finding the condition for equality took another quarter of a century, being due to Peter J. Cameron and Cheng Yeaw Ku, 2003 and, independently, Benoit Larose and Claudia Malvenuto, 2004.

Web link: www.maths.qmul.ac.uk/postgraduate/anncook/ku04.pdf

Further reading: *Groups and Geometry* by Peter M Neumann, Gabrielle A Stoy and Edward C Thomson, OUP, 1994.