**THEOREM OF THE DAY**

**Bayes’ Theorem** Suppose a sample space $S$ is partitioned into two non-empty parts $B_1$ and $B_2$. Then the conditional probability that a point in $S$ satisfying some property $A$ will also lie in $B_1$, is given by

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}.$$

A farmer has herded all her thirty cows into two fields, imaginatively called $B_1$ and $B_2$. The black cows have conveniently migrated to the bottom of each field and form a subset $A$ of the cows. Suppose we find a cow, what can we say about which field we are in? The probability of a cow being in field $B_1$, $P(B_1)$, is $10/30 = 1/3$ and $P(B_2) = 20/30 = 2/3$. Now suppose that the cow we found was a black cow, how does this affect the odds that we are in field $B_1$? The probability of the cow we found being black, given that we are in field $B_1$, is $P(A|B_1) = 3/10$, since 3 of the 10 cows in $B_1$ belong in $A$; similarly $P(A|B_2) = 8/20 = 2/5$. Bayes’ theorem tells us that $P(B_1|A) = \frac{3}{10} \cdot \frac{1}{3} \left( \frac{3}{10} \cdot \frac{1}{3} + \frac{2}{5} \cdot \frac{1}{3} \right) = \frac{3}{11}$; clearly correct, since 3 out of the 11 black cows are in field $B_1$. So the chance we are in $B_1$ drops from $\approx 33\%$ to $\approx 27\%$.

The theorem extends to a greater number of regions, $B_1, B_2, B_3, \ldots$, by extending the sum in the denominator in the obvious way; and to a single region, $B_1$, by replacing the denominator by $P(A)$. The Rev. Thomas Bayes was one of the first to write about conditional probability. His work was published after his death in 1761 but lay for a long time forgotten. His theorem has eventually come to be used controversially to convert possibly subjective assessments of prior probability $P(B_1)$ into a posterior probability $P(B_1|A)$. This so-called Bayesian approach has sometimes been accused of applying the rigorous machinery of probability theory to inputs which may be guesswork or supposition.

Web link: [www.dcs.qmul.ac.uk/~norman/papers/probability_puzzles/cancer.html](http://www.dcs.qmul.ac.uk/~norman/papers/probability_puzzles/cancer.html).