THEOREM OF THE DAY

Distribution of local maxima in random samples *Let* $\pi = (\pi_1, \ldots, \pi_n)$ *be a permutation of* $\{1, \ldots, n\}$ *and let* k *be a positive integer. The* k*-local maxima of* π *are defined to be the maximum values taken by length* k *subsequences of* π*: i.e., the set* $\{\max(\pi_i, \ldots, \pi_{i+k-1}), 1 \leq i \leq n-k+1\}$. *Denote by* $f_k(n,m)$ *the number of permutations* π *having exactly* m *distinct* k*-local maxima. Let* $v_k(x,y)$ *be the generating function for the* f_k *defined as* $v_k(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} f_k(i,j)x^i y^j/i!$. *Then the discrete distribution of the probabilities* $\Pr(\pi$ *has* m *local maxima),* $m = 0, \ldots, n$*, is given by*

$$\frac{1}{n!}\frac{\partial^n v}{\partial x^n}\bigg|_{x=0}.$$

Moreover, $v_k(x,y)$ *satisfies the partial differential equation*

$$\frac{\partial v}{\partial x} = yv^2 + (1-y)\left(1 + 2x + \ldots + (k-1)x^{k-2}\right), \tag{1}$$

with boundary conditions $v_k(0,y) = \frac{\partial v}{\partial x}\big|_{x=0} = 1$.

This theorem is ready for uploading but is being checked for correctness/copyright. If you would like to review it in advance please contact me: comments@theoremoftheday.org.